Informatics 1 Cognitive Science – Tutorial 5 Solutions

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Week 6

In class we discussed categorization and how conceptual knowledge is organized. The aim of this tutorial is to understand theories of categorization and relations between categories. The second part of the tutorial deals with judgement and decision making.

1 Classial Concepts

This exercise is designed to shed light on some challenges facing classical (or definitional) accounts of concepts and some phenomena that are relevant to theories of concepts and categories.

Under classical accounts, concepts are represented using sets of necessary and jointly sufficient properties. For instance, we might say that a triangle is a polygon with three vertices. If something isn't a polygon, or if it doesn't have three vertices, it isn't a triangle (necessity)[1]. If it has both of these properties, it must be a triangle (joint sufficiency). In the case of mathematical concepts, this can work fairly well, but we'll see it's a bit more difficult for everyday objects and events. If you want a trickier task, replace the word "rabbit" below with "art", or "game". Here's the exercise:

1. Get a sheet of paper (or text editor) and write down (as best you can) the necessary and jointly sufficient properties something must have to be a **rabbit**. When you're done, pass your list to the person on your left (or below you in the tutorial chat).
2. When you receive the list from someone else, come up with a counterexample. This is either (1) something you would call a rabbit, which does not have one of the listed properties, or (2) something that isn't a rabbit but has all of the properties on the list. Update the list of properties to account for your counterexample.
3. Pass the updated list to the next person and repeat the process from step 2. When your original list comes back to you, you're done, at which point it's time to discuss the lists.

Discussion

Compare your lists and discuss the following questions.

- Are the lists effectively the same – do they pick out the same sets of things as being rabbits?
- For each list, what's the last counterexample that someone came up with? Does everyone agree that this counterexample is, or isn't, a rabbit?
- Are there counterexamples where people agree that something is a rabbit, but disagree about the extent to which it's a good, or typical example of a rabbit?
- Does everyone agree that the lists (or some of them) are complete, and admit no new counterexamples?

[1]Naturally, we'll need to define vertices and polgyons as well

Solution There aren't right and wrong answers in this exercise, but hopefully it'll be a useful medium for discussing the following points (and others):

- It's difficult to define concepts in terms of necessary and sufficient properties. This doesn't logically imply that our mental representations aren't definitional, but it nonetheless bodes ill for the theory.
- Concept membership tends to be graded. A penguin and a robin are both birds in almost everyone's assessment, but people show other behaviors suggesting that a robin is a better, more prototypical example of a bird than a penguin is. The classical theory of concepts needs to introduce additional machinery to explain this.
- Not everyone agrees about what goes into a particular category. This isn't a problem for definitional theories in particular – people might vary in their mental representations whatever they are – but it's good to keep in mind when assessing theories of concepts and explanations of how we learn or construct concepts, and to think about how variability in people's concepts might have implications for communication.

2 Similarity-based Concept Models

Imagine you've just been introduced to two new categories for the first time: "daxes" and "grobs". As far as you can tell, daxes and grobs are all identical except for their weights. A person gives you 100 daxes that weigh around 10 kg, and 100 daxes that weigh around 100 kg. The person also gives you 200 grobs that have widely varying weights between 1 kg and 200 kg, with an average of weight of 105 kg.

You now find two new objects (object P and object Q). P weighs 101 kg, and Q weighs 57 kg.

1. According to prototype theory as discussed in lecture, is P more likely to be a dax or a grob? Why?

   Solution: The average dax weight is around 55kg, while the average grob is 105kg. At 101kg, P is closer to the average grob, so it's more likely to be a grob.

2. According to prototype theory as discussed in lecture, is Q more likely to be a dax or a grob? Why?

   Solution: The average dax weight is around 55kg, while the average grob is 105kg. At 57 kg, Q is closer to the average dax, so it's more likely to be a dax.

3. According to exemplar theory as discussed in lecture, is P more likely to be a dax or a grob? Why?

   Solution: P is close in weight to a large number of dax exemplars so we might expect that it's a dax.

4. According to exemplar theory as discussed in lecture, is Q more likely to be a dax or a grob? Why?

   Solution: Q is far from most dax exemplars and likely to be comparatively close to several grob exemplars, so we might expect it's a grob. That said, it depends on one's weighting function, and students haven't been exposed to classical weighting functions (which tend to involve exponential decay).

3 Wason's selection task

In lecture, we discussed Wason's card selection task, and how we can understand the typical person's decision in this task in different ways.

The classic version of the task asks participants to decide which cards to flip over to evaluate whether a rule is true or not. For example, the cards might have letters on one side and numbers on the other, and the rule might be "if there is a vowel on one side of a card, there is an even number on the opposite side." If the cards' visible faces are "I", "R", "2", and "3", which cards would you flip over?

Now consider the following scenarios, both of which are similar to the classic task: They have a rule of the form "if P then Q", and elements analogous to the cards.

A. A geneticist has discovered the ARGH gene. She conjectures that if an organism has the ARGH gene, it must be a mammal. She has tested many genetic samples (of unknown origin), and found that around 99 percent contain the ARGH gene. She can test new organisms for the ARGH gene, or look up the species associated with one of her previous ARGH-gene tests.

B. A local apple market requires that people have green eyes to buy apples; people with other eye colours are not permitted to carry apples. Imagine you're playing the role of market rule enforcer.

Exercises

1. For each of these scenarios, what four "cards" correspond to those in the classic task? What action is analogous to flipping over a card in each scenario?
   Solution: (1) P and not-P: has-ARGH and lacks-ARGH; looking up the species corresponds to flipping these. Q and not-Q: mammal and not-mammal; testing for ARGH corresponds to flipping these.
   (2) P and not-P: A person carrying more than zero apples and a person carrying zero apples (each wearing sunglasses, perhaps). Q and not-Q: a green-eyed person and a not-green-eyed person (each with opaque bags, perhaps).

2. Which of these "cards" do you feel you should "flip over", intuitively?
   Solution: This is a matter of personal preference and intuition, but both of these variations resemble real experiments where people tend to choose P and not-Q, contra Wason's claim that the orginal results are attributable to people being bad at "formal operations".

3. Do these judgments agree or disagree with your judgments in the original task? If they disagree, reflect on why that might be.
   Solution: There aren't any right answers here, but hopefully students will independently arrive at some of that same conclusions that motivated follow-ups to Wason's original task. For example, Oaksford and Chater argued that people tend to choose P and Q because those cards are informative when cases of P and Q are generally rare (their detailed arguments rest on ideas from information theory that are outside the scope of examinable material, but the references from lecture may interest some keen students). Here, P is very common, suggesting that P and not-Q would be preferred to P and Q under an uncertainty-minimising view. Thus, "passing" this case (according to Wason's criteria, i.e., P and not-Q) while "failing" the standard one comports with O&C's view of the card selection task. The apple-market example frames the task in terms of enforcing rules or norms; some authors (e.g., Fiddick et al., Cosmides and Tooby) have argued that people are sensitive to violations of rules and social norms in ways that allow them to "pass" the card-selection task if it's expressed in this way.

4 Lotteries

We discussed lotteries in the context of framing effects and prospect theory. When reasoning about the subjective value of decisions in an uncertain environment, it is common to focus on **expected utility,** or the sum of the utilities of different possible outcomes weighted by their probabilities. For example, if we assign a utility of 10 to a coin coming up heads, and a utility of 5 to the coin coming up tails, and believe a heads will come up with probability 0.6, then the expected utility is $0.6 \times 10 + 0.4 \times 5 = 8$. In the following example, we will focus first on monetary gains and losses, rather than utilities.

Suppose we have a lottery in which there are 1000 tickets printed with unique numbers. After all of the tickets have been purchased, one of the ticket numbers will be called out randomly (all are equally likely), and the person with that ticket will win £500.

Exercises

1. What is the expected monetary gain or loss for someone who purchases one ticket for £1?
   Solution: If all tickets are equally likely to win, then the probability of winning is 1/1000. The expected reward is
   $$\sum_{i=1}^{N} R(o_i)P(o_i)$$
   where $N = 2$, o_1 is the outcome of winning, $R(o_1) = 500 - 1$, $P(o_1) = 0.001$, $R(o_2) = -1$, and $P(o_2) = 0.999$. That comes out to $0.499 - 0.999 = -0.5$.

2. One way to assess whether a person takes a rational approach to decision-making is to see if there are situations where their beliefs lead to guananteed, avoidable losses. Consider a person who thinks £1 is a fair ticket price for buying or selling a ticket and is willing and able to buy any number of tickets. Is there a way for someone to take this person's money without incurrent any risk?
   Solution: Yes. In this case, the lottery operator can make guaranteed money by selling at least 501 tickets to the person. If the person is sold **all** of the tickets, the operator makes a profit of £500: £1000 guaranteed ticket revenue minus £500 from paying out on the lottery.

3. What if this person is only willing to buy a single ticket?
   Solution: Here the bookie cannot guarantee the person will lose money, so it isn't risk free. This suggests there are utility functions where this value assignment could make sense.

4. Under what circumstances, if any, might it be rational for a person to purchase a single ticket for £1, considering only selfish economic concerns, e.g., setting aside any pleasure a person might take in gambling and the possibility that ticket proceeds go to charity? Feel free to use contrived scenarios if necessary.
   Solution: Sometimes a person's utility can be a non-linear function of monetary gains or losses. For example, if a person is starving and lives in a society where food is unavailable for £1, then that person might reasonably assign a utility of zero to any amount of money that doesn't prevent starvation. In that case, a small chance of winning enough money to buy food is better than the alternative.