WEEK 8

The "relevant" logic R, designed to avoid paradoxes of material implication, is most conveniently formulated by allowing assumptions to be combined in two ways, so that assumption numbers may be separated by commas (,) or by semicolons (;). The intuitive idea is that

A,B entails C

iff A and B cannot both be true and C false, while

A;B entails C

iff A gives us a licence to infer C from B.

Some of the rules have to be stated carefully.

$$\frac{X : A \qquad\qquad Y : B}{X,Y : A \,\&\, B}\ (\&I)$$

note the comma.

$$\frac{X : A \to B \qquad\qquad Y : A}{X;Y : B}\ (MPP)$$

note the semicolon.

$$\frac{X;A : B}{X : A \to B}\ (CP)$$

note the semicolon.

As in Lemmon's orthodox system, within bunches of a uniform type (all "," or all ";") order and repetitions don't matter. So we can allow

$$\frac{3 : P \qquad\qquad 3 : Q}{3 : P \,\&\, Q}\ (\&I)$$

for instance. We do not, however, allow

$$\frac{1;3 : P \qquad\qquad 2;3 : Q}{(1,2);3 : P \,\&\, Q}\ (?)$$

as an application of &I, because it is not of the given form - &I would give

(1;3),(2;3) : P & Q

which is a different sequent. The following proof illustrates the rules.

1	(1)	P → Q	A	
2	(2)	Q → R	A	
3	(3)	P	A	
1,2	(4)	(P → Q) & (Q → R)	1,2 &I	
1,2	(5)	P → Q	4 &E	{ This unifies the L.H.S.
1,2	(6)	Q → R	4 &E	of the two conditionals }
(1,2);3	(7)	Q	3,5 MPP	
(1,2);3	(8)	R	6,7 MPP	{ Two ocurrences of (1,2)
				are treated as one here }
1,2	(9)	P → R	3,8 CP.	

The other rules which have to be modified are MTT, RAA and vE. MTT and RAA are easy.

$$\frac{X \;:\; A \to B \qquad\qquad Y \;:\; -B}{X;Y \;:\; -A} \text{ (MTT)}$$

(semicolon)

$$\frac{X;A \;:\; B \,\&\, -B}{X \;:\; -A} \text{ (RAA)}$$

(semicolon)

For vE we need the idea of two bunches of assumptions, built up using commas and semicolons, which are exactly the same except that one has a formula A where the other has B. We use the notation X[A] and X[B] for these. For example, if A and B are the formulas P and Q respectively, and X[A] is

R&S , (P ; R)

then the corresponding bunch X[B] is

R&S , (Q ; R).

By extension, we can talk of the result of putting a whole bunch, Y, of assumptions in place of A, getting X[Y]. In the above example, if Y were the bunch P;(Q,R) then X[Y] would be

R&S , ((P;(Q,R) ; R).

Now we can state vE in its full generality:

$$\frac{X \;:\; A \lor B \qquad\qquad Y[A] \;:\; C \qquad\qquad Y[B] \;:\; C}{Y[X] \;:\; C} \text{ (vE)}$$

Strictly speaking, this is not yet quite enough to specify the system R, for we should make precise the convention that "order and repetitions don't matter". This we do by stipulating that bunches of assumptions of certain forms are equivalent and can replace each other anywhere in proofs. The equivalences are as follows.

X,Y equivalent to Y,X
X;Y equivalent to Y;X
X,(Y,Z) equivalent to (X,Y),Z
X;(Y;Z) equivalent to (X;Y);Z

In addition, in any position,

X may replace X,X
X may replace X;X.

A bunch of the form X;Y may <u>not</u> have one side empty, so it is convenient to add a "dummy" bunch £ (for "£ogic") and specify its characteristic property:

£;X equivalent to X.

These details are given only to fill out the account of system R. They are for information only: you are not required to memorise them (though if you did, that would be splendid!). You may, however, be invited to construct proofs in R using the "→" and "&" rules, so don't throw this sheet away.