WEEK 6

We may draw out a tree structure with a formula at each node, in order to test a formula (or set of formulas) for <u>consistency</u> or <u>inconsistency</u>.

Begin by writing down the formula(s) to be tested, in a vertical column if there are two or more.

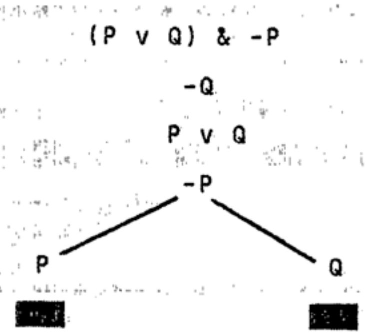
Each node (formula) in the tree other than an atom or a negated atom must be <u>developed</u> in every branch leading down from it.

A conjunction A & B is developed by writing the conjuncts one below the other. This represents the point that for the conjunction to be true both the conjuncts must be true as well. A disjunction A v B is developed by splitting the branch into two, one continued with A and the other with B. So the disjunction survives iff one or other disjunct survives.

A branch of the tree <u>dies</u> if it contains a <u>contradiction</u>: a pair of nodes of the form A and -A. The tree dies iff every branch dies. In that case, there is no way the formula(s) being tested could (all) be true.

As an example, consider the two formulas (P v Q) & -P, -Q. Could they both be true together? Let us see.

1.
2.
3.
4.



developing line 1

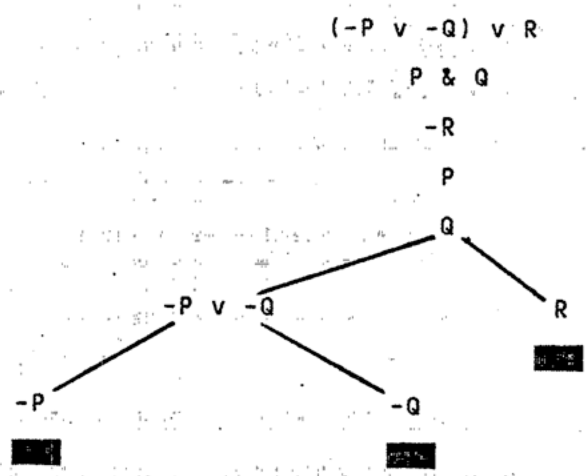
developing line 1

developing line 3

contradictions lines 4 and 5 on the left, lines 2 and 5 on the right.

As another example, consider (-P v -Q) v R, P & Q, -R.

1.
2.
3.
4.
5.
6.
7.



from 2

from 2

from 1

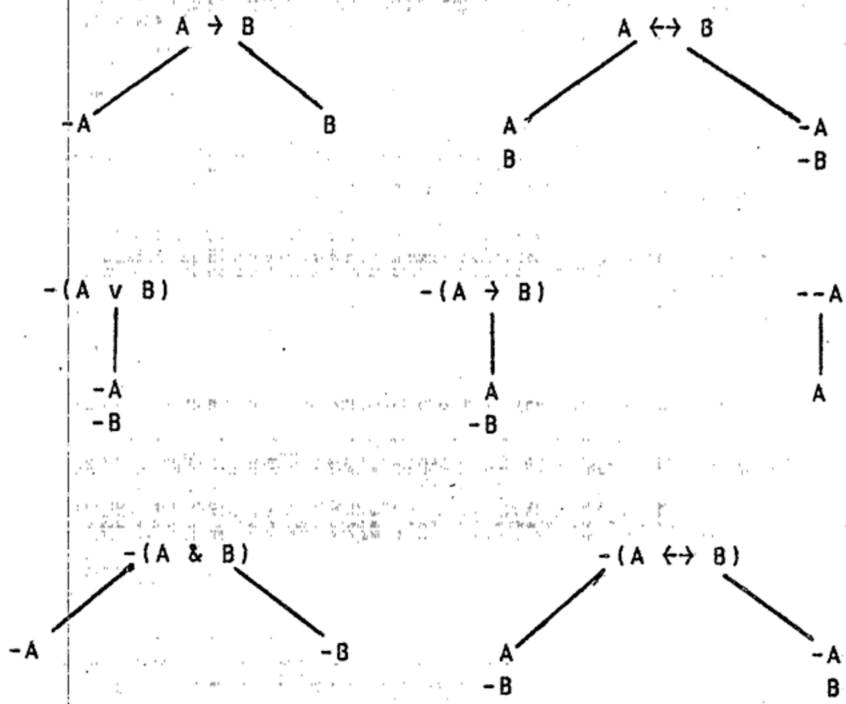
3, 6 on the right

from 6

4, 7 on the left

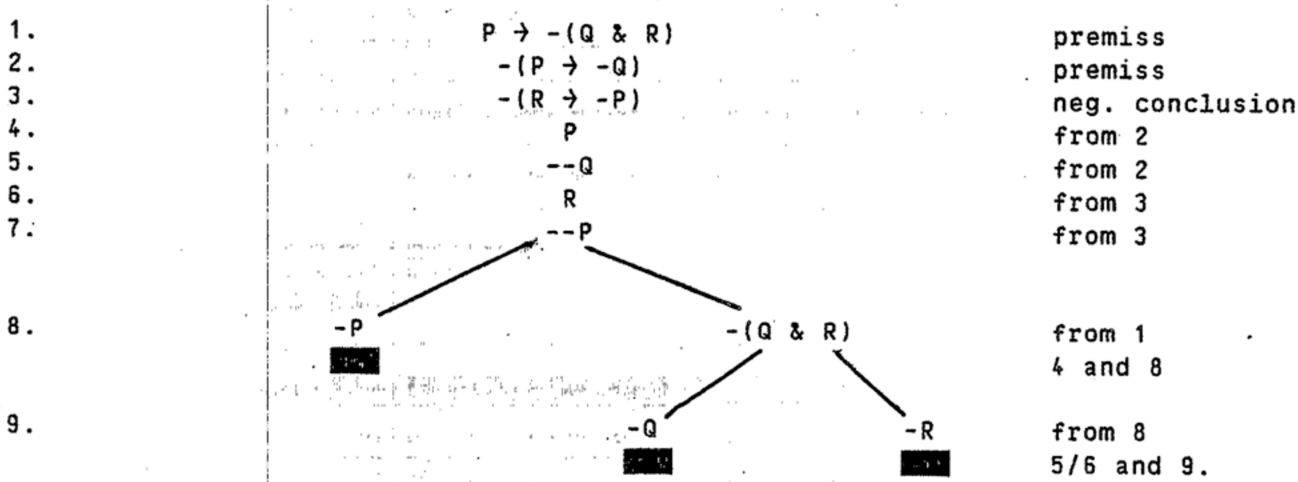
5, 7 on the right

Rules for other connectives and negated cases



To test a sequent $A_1 \ldots\ldots A_n$: B for validity, see whether there is any way all the premisses could be true and the conclusion false. That is, test the set { $A_1 \ldots\ldots A_n$, -B } for consistency. Suppose all the premisses true and the conclusion, B, false. Develop the truth tree by the rules given above. If all branches die because of contradictions, the sequent is valid. For instance, let us test

P → -(Q & R), -(P → -Q) : R → -P



All the branches eventually contain contradictions, so the sequent is valid. To produce a proof from primitive rules in Lemmon's formulation of logic would be rather difficult, but the truth tree method handles it very easily.