

Thursday and Friday tutorial groups: hand in solutions at your next tutorial (in week 9). Monday and Wednesday groups: you need not hand in solutions to this sheet.

Prove the following sequents. TI and SI may not be used.

1. ✓ $(x)Fx \& (y)Gy \vdash (z)(Fz \& Gz)$
2. ✓ $(x)Fx \vee (x)Gx \vdash (x)(Fx \vee Gx)$ { vE proof }
3. ✓ $(x)(Fx \rightarrow Gx), (x)(Hx \rightarrow -Gx) \vdash (x)(Fx \rightarrow -Hx)$
4. ✓ $(x)(Fx \rightarrow Gx) \vdash (x)(y)((Fx \& -Gy) \rightarrow (Gx \& -Fy))$
5. ✓ $(y)(Gy \rightarrow Hy) \vdash (\exists x)Gx \rightarrow (\exists y)Hy$ ^H
6. ✓ $(\exists x)(Fx \& Gx) \vdash (\exists x)(Gx \& Fx)$
7. $(\exists x)Fx \vdash -(\exists x)-Fx$
8. $(\exists x)(Fx \& -Gx) \vdash -(\exists x)(Fx \rightarrow Gx)$

These constitute LEMMON-AID's sheet E5. Sheet S5 is well worth doing for practice, as it gives you mostly simpler examples. You must become familiar with proofs involving quantifiers, so it will pay to work at all the above proofs and sheet S5, even if you are in a Monday or Wednesday tutorial group. There will certainly be an exam. question asking for a proof of this type.

- $$(\exists x)Fx \vdash -(\exists x)-Fx$$
- | | | | |
|------|-----|-------------------|------------|
| 1 | (1) | $(\exists x)Fx$ | A |
| 2 | (2) | $(x)-Fx$ | A |
| 2 | (3) | $-Fa$ | 2 vE |
| 4 | (4) | Fa | A |
| 2, 4 | (5) | $Fa \& -Fa$ | 3, 4 & I |
| 4 | (6) | $-(x)-Fx$ | 2, 5 RAA |
| 1 | (7) | $-(\exists x)-Fx$ | 1, 4, 6 EE |