WEEK 10

Sortal Quantifiers

Instead of expressing 'All philosophers are generous' by means of a variable ranging over "things" and a connective (→) inside the scope of a quantifier, it would be simpler and more convenient to have variables just ranging over philosophers. We could use lower case letters f, g, h...... to range over the Fs, the Gs, the Hs..... in a straightforward way. So instead of

(x)(Fx → Gx) "Every thing is an if-philosopher-then-generous"

we can write

(f)Gf "Every philosopher is generous".

In a similar way, "Some philosophers are generous" would become

(∃f)Gf

instead of the more cumbersome

(∃x)(Fx & Gx) "Some thing is a philosopher-and-generous".

With this notation, it becomes easier to formalise complicated examples. Consider, for example, "Every philosopher likes some but not all jokes":

(f)[(∃j)Lfj & -(j)Lfj]

instead of the standard version

(x)(Fx → [(∃y)(Jy & Lxy) & -(y)(Jy → Lxy)]).

The rules for introducing and eliminating these "sortal" quantifiers in proofs are slightly more complicated than the usual ones. In stating them, I shall let 'P' stand in for any (monadic) predicate letter and let 'p' stand in for the corresponding variable. Terms are as usual. There is a special predicate letter 'E' (for 'exists') with special variables 'x', 'y', 'z' corresponding to it exactly as in the familiar case. Now:

UE.

$$\frac{(p)A[p] \qquad Pt}{A[t]}$$

where term t gets substituted for sortal variable p in A[p] as usual.

UI.

$$\frac{X, Pt \ : \ A[t]}{X \ : \ (p)A[p]}$$

where the substitution is again as usual, and t does not occur in any formula in X.

Note that there is an extra input line for UE. This is necessary, since if t were not a P then its being an A would not follow from 'All Ps are A'. UI now discharges the assumption Pt. For UI purposes, t is not just an arbitrarily chosen <u>object</u>; it is an arbitrarily chosen <u>P</u>.

EI. $$\frac{A[t] \qquad Pt}{(\exists p)A[p]}$$ substitution of t for p again as in the standard case.

EE. $$\frac{X \;:\; (\exists p)A[p] \qquad Y,\ Pt,\ A[t] \;:\; B}{X,\ Y \;:\; B}$$ where, as usual, t does not occur in Y or in B.

The extra input line for EI is necessary because it does not follow merely from the fact that, say, Tebbitt is alarming that some <u>philosopher</u> is alarming. In applying EE, we discharge not merely the assumption that t is an arbitrary A but the assumption(s) that t is an arbitrary P which is A.

Everything exists. Although formally the logic of sortals is rather like free logic, we want to be orthodox at this point so we add a special rule governing the predicate symbol 'E'. We shall call it the "Rule of Existence", and annotate it with the expression 'EX'.

$$\frac{}{Et}$$

That is, Et may be introduced at any point, for any term t, with no assumption numbers at all. No line numbers are cited.

Here are a couple of proofs, to illustrate the rules for sortal quantifiers.

(f)Gf, (g)Hg ⊢ (f)Hf

1	(1)	(f)Gf	A	
2	(2)	(g)Hg	A	
3	(3)	Fa	A	{ Needed for UE purposes }
1,3	(4)	Ga	1,3 UE	
1,2,3	(5)	Ha	2,4 UE	
1,2	(6)	(f)Hf	3,5 UI	{ Assumption 3 discharged }

(∃f)Gf ⊢ (∃x)(Fx & Gx)

1	(1)	(∃f)Gf	A	
2	(2)	Fa	A	
3	(3)	Ga	A	
2,3	(4)	Fa & Ga	2,3 &I	
	(5)	Ea	EX	{ Needed for EI }
2,3	(6)	(∃x)(Fx & Gx)	4,5 EI	
1	(7)	(∃x)(Fx & Gx)	1,2,3,6 EE.	{ From lines (1) and (6) discharging 2 and 3 }