

5. Prove the following. You may *not* use TI or SI:

$$(a) (\exists x) (y) (Fy \rightarrow Rxy) \mid - (x) (Fx \rightarrow (\exists y) Ryx)$$

$$(b) (\exists x) (y) (Fy \rightarrow x=y) \mid - (x) (y) ((Fx \& Fy) \rightarrow x=y)$$

$$(c) (x) (y) ((Fx \& Fy) \rightarrow x=y) \mid - (\exists x) (y) (Fy \rightarrow x=y)$$

6. 'The most striking feature of the language of formal logic is its extreme poverty compared with the richness of natural language idiom.' Illustrate this claim. What should we conclude from it?

7. EITHER: How may Lemmon's system of logic best be amended to avoid validating 'paradoxes of material implication'?

OR: How may Lemmon's system of logic best be defended against the charge that 'paradoxes of material implication' show it to be incorrect?

N B

(a)

Write out the truth table for the formula

$$((P \vee Q) \rightarrow R) \leftrightarrow \neg(P \& \neg R).$$

SECTION B

3. For both parts of this question, use the lexicon provided.

(a) Give reasonably natural English readings of the following formulas:

$$(i) (x) (\exists y) (-Ry \& Bxy)$$

$$(ii) (\exists x) (x \neq m \& (y) (-Ry \rightarrow Byx))$$

$$(iii) (\exists x) ((y) (Bmy \leftrightarrow x = y) \& Rx)$$

$$(iv) (x) (y) (\exists z) ((z \neq x \& z \neq y) \& Bzm)$$

(b) Formalise these sentences, adding comments on any particular difficulties you discern:

(i) Morton is not the only side to be relegated.

(ii) Any team can beat any other team that Morton can beat.

(iii) A team that can beat no team that can beat Morton is relegated.

(iv) There is exactly one team capable of losing to Morton.

Lexicon. Bxy : x can beat y

Rx : x is relegated

m : Morton.

4. (a) Use truth tables to decide the validity of:

$$\checkmark (i) P \rightarrow \neg Q, P \vee Q \models \neg(Q \& \neg P)$$

$$\checkmark (ii) P \rightarrow (Q \vee R), Q \rightarrow \neg S \models ((S \& P) \rightarrow R) \& ((S \rightarrow \neg P) \vee (P \rightarrow R))$$

(b) Consider adding a new connective, $*$, to the system of logic. It is to have the truth table:

A	B	A*B
T	T	F
T	F	F
F	T	F
F	F	T

(i) Find a formula using just $\&$ and \neg with the same truth table as $P*Q$.

(ii) Find formulas using only $*$ with the same truth tables as $\neg P$, $P \vee Q$, $P \& Q$ and $P \rightarrow Q$.

Solutions to be HANDED IN at your next tutorial (week 4 for Monday and Wednesday groups, week 3 for Thursday and Friday groups).

Prove the following sequents, using the rules A, &I, &E, MPP, CP.

1. $Q \& R, P \vdash (P \& Q) \& R$ ✓
2. $Q \rightarrow R, P \rightarrow Q \vdash P \rightarrow R$
3. $P \rightarrow Q, P \rightarrow R \vdash P \rightarrow (Q \& R)$ ✓
4. $P \rightarrow Q \vdash (P \& R) \rightarrow (Q \& R)$ ✓
5. $P \rightarrow (Q \rightarrow R) \vdash (P \& Q) \rightarrow R$?
6. $(P \& Q) \rightarrow R \vdash P \rightarrow (Q \rightarrow R)$?
7. $P \rightarrow (Q \rightarrow R) \vdash Q \rightarrow (P \rightarrow R)$?
8. $P \rightarrow (Q \rightarrow R) \vdash (S \rightarrow Q) \rightarrow (P \rightarrow (S \rightarrow R))$ ✓

These sequents constitute sheet E1 for the computer program LEMMON-AID. If you wish, you may use a micro to help you get correct proofs. If you store the completed proofs on a LEMMON-AID disk, this may be handed in instead of the written version. Floppy disks should be handed to me (JKS) rather than to your tutor. LEMMON-AID sheets S1 and X1 contain more exercises using the same rules. These could be used for practice.