Answers to this sheet <u>may</u> be handed in to your tutor, but <u>need not</u> be.

Prove the following sequents, using all the primary rules including =I and
=E, but not secondary rules (TI and SI).

1. $\vdash$ (x)(∃y)(x = y)

2. Fa $\vdash$ (∃x)(Fx & x=a)

3. (∃x)(Fx & x=a) $\vdash$ Fa

4. (x)(Fx → x=m) $\vdash$ (∃x)Fx → Fm

5. (x)(Fx → (Gx v x=m)) $\vdash$ Gm → (x)(Fx → Gx)

6. (∃x)(Fx & Gx), (∃x)(Fx & -Gx) $\vdash$ -(x)(y)((Fx & Fy) → x=y)

7. (∃x)(y)(Fy → x=y) $\vdash$ (x)(y)((Fx & Fy) → x=y)

8. (x)(y)((Fx & Fy) → x=y) $\vdash$ (∃x)(y)(Fy → x=y)

These sequents constitute LEMMON-AID's sheet E7.
As usual, sheet S7 has some more useful examples and sheet X7 has yet more.
Note that LEMMON-AID likes to write '-(x = y)' as 'x ≠ y', but if you are
typing in formulas you have to write negated identities in full.

IMPORTANT NOTICE

There will be some informal <u>revision classes</u> for Logic 1h during the Summer
Term. Information about these will be posted on the board in the Philosophy
Department in week 1 of next term. Please go along and check the details at
that time if you are interested in taking part in such classes.

LEMMON-AID will continue to be available for revision purposes throughout the
academic year. Sign out copies of the disk as usual.

Solutions to the _starred_ exercises (and to those only) to be HANDED IN at your first tutorial in the Spring Term.   Please regard this deadline as strict.

Tutorials will continue until week 3 of next term, at the same times as they have been this term.   If this causes problems, see your tutor or me (JKS) before missing classes.

A.   Prove the following, without using TI or SI.

     1.   -(∃x)Fx   ⊢   -Fa   { use EI and MTT }

*** 2.   -(∃x)Fx   ⊢   (x)-Fx   { note stars: hand in }

     3.   -(∃x)-Fx   ⊢   (x)Fx

     4.   -(P v Q)   ⊢   -Q   { vI and MTT }

     5.   -(P v (∃x)-Fx)   ⊢   (x)Fx   { see 3 and 4 above }

*** 6.   ⊢   (x)Fx v (∃x)-Fx

     7.   -P   ⊢   P → Q

*** 8.   (∃x)-Fx   ⊢   (∃x)(Fx → P)

     9.   -(Fa & -P)   ⊢   Fa → P   2.2.5(5).

    10.   -(∃x)(Fx → P)   ⊢   Fa & -P

*** 11.   ⊢   (∃x)(Fx → (y)Fy)

B.   Prove these, also without using TI or SI.

     1.   (y)Rmy   ⊢   (∃x)Rxm

     2.   (∃x)(y)Rxy   ⊢   (∃x)Rxa

*** 3.   (∃x)(y)Rxy   ⊢   (y)(∃x)Rxy

     4.   (x)(y)Rxy   ⊢   (y)(x)Rxy

*** 5.   (∃x)(∃y)Rxy   ⊢   (∃y)(∃x)Rxy

*** 6.   (x)(y)(Rxy → Ryx)   ⊢   (x)((∃y)Rxy → (∃y)Ryx)

*** 7.   (x)(Fx → Gx)   ⊢   (x)((∃y)(Fx & Rxy) → (∃y)(Gy & Rxy))

C.   Give counterexamples to the following sequents.

1.   (x)(Fx → Gx)   :   (x)(-Fx → -Gx)

2.   (x)(Fx v Gx)   :   (x)Fx v (x)Gx

3.   (∃x)(Fx → Gx)   :   (∃x)Fx → (∃x)Gx

4.   (x)(Fx → Gx), (∃x)(Gx & Hx)   :   (∃x)(Fx & Hx)

5.   (x)(∃y)Rxy   :   (∃y)(x)Rxy

6.   (x)(y)(Rxy → Ryx)   :   (∃x)Rxx

D.   <u>Challenge</u>.

It happened, in the days when trains used to call at the tiny village of Much
Tittering in the Woods, that the 1215 once pulled up there and stood for the
best part of an hour.   Nobody now remembers why.   At any rate, the driver,
the porter, the ticket instector, the stationmaster and the guard spent the
time in such merry conversation as is customary among employees of railway
companies.   Their names, in alphabetical order, were James, Kant, Locke, Mill
and Neitzsche.   For reasons lost in the mists of railway history, they agreed
to make two statements each, one true and the other false.   They said:

MILL: Neitzsche is the stationmaster.   F
Station master   James is the guard.   T

KANT: Mill's second statement was false.   F
Porter   The porter's favourite snack is red herring. T

LOCKE: Neitzsche is not the ticket inspector.   T
Ticket Insp.   Mill is not the stationmaster.   F

NEITZSCHE: Locke's first statement was false.   F
Driver   Kant is the porter.   T

JAMES: I am not the ticket inspector.   T
Guard   Neitzsche's second statement was false.   F

What was the driver's name?

E.   A LOGICAL CHRISTMAS TO ALL OUR READERS.