SHEET 2

Comment on the following arguments.   Say whether they are valid or invalid,
and point out the difficulties you find them to raise.

1.   If it rains I shall get wet;  but if I take my umbrella I shall not get wet.

Therefore, if I take my umbrella it won't rain.

2.   All animals are either vertebrate or invertebrate.
All mammals are vertebrate.
Raquel is obviously a mammal.

Therefore she is an animal.

3.   Nothing is better than Château Mouton Rothschild, but even Newcastle Brown
is better than nothing.

Therefore, Newcastle Brown is better than Château Mouton Rothschild.

4.   Every even number (greater than 2) is the sum of two prime numbers.
82  =  23 + 59.
23 is prime, and 59 is prime.

Therefore 82 is even.

Prove the following:

1.    P    ⊢    P & P

2.    ⊢    (P → Q) → ((Q → R) → (P → R))

3.    ⊢    ((P → P) → Q) → Q

4.    Q    ⊢    P → Q

5.    Construct, either on an "assembly line" or in tree form, the formula

      (-P & Q) → (P v --(R → (Q & Q))).

6.    The following is a proof with some bits missing.

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      1     (1)  | -(P v ((P v Q) → Q))         A
      2     (2)  | P                            A
      2     (3)  | P v Q                        2 vI
            (4)  | Q                            2,3 CP
      1     (5)  | -P                           1,4 MTT
      6     (6)  | -Q                           A
            (7)  |                              2,6 &I
            (8)  |                              7 &E
            (9)  |                              5,8 &I
            (10) |                              6,9 RAA
      1,2   (11) | Q
      12    (12) | P v Q                        A
            (13) |
            (14) |                              12,2,11,13,13 vE
            (15) | (P v Q) → Q                  12,14 CP
      1     (16) |                              15 vI
            (17) |                              1,16 &I
            (18) |                              1,17 RAA
            (19) | P v ((P v Q) → Q)
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      (a)   Fill in the gaps.

      (b)   Write out the sequents represented by lines (5), (9) and (19).