Exercise 3

Form all critical pairs from the rewrite rule:

$$h(h(x)) \Rightarrow k(x)$$

Apply the rule exhaustively to the elements of the critical pairs, and see which pairs conflate. Suggest any new rule(s) which might be added to make a confluent set.

need 1 other rule.
$h(k(x)) \Rightarrow k(h(x))$.

Exercise 4

Here are three axioms from group theory:

1. $e.a = a$ (There exists a left identity e)
2. $a^{-}.a = e$ (Every element a has a left inverse w.r.t. e)
3. $(a.b).c = a.(b.c)$ (Multiplication is associative)

Taking these as rewrite rules ordered left to right, use the Knuth-Bendix procedure to discover the rest. Notice that as new axioms are derived, older ones may no longer be irreducible and they should be superseded by a version which cannot be immediately rewritten by one of the other rules. This is not a short exercise.

need 7 more rules for this example.

Exercise 1

Draw the LUSH resolution search space for these clauses, using 6 as the top clause:

home clauses

$$
\begin{aligned}
p(X) \wedge q(X) &\rightarrow r(X) & (1)\\
s(X) &\rightarrow r(X) & (2)\\
t(X) &\rightarrow s(X) & (3)\\
&\rightarrow p(a) & (4)\\
&\rightarrow q(a) & (5)\\
r(a) &\rightarrow & (6)
\end{aligned}
$$

Give an interpretation which could be used to control this search, and explain its effect on the search space.

Exercise 2

Apply Boyer and Moore's techniques to prove that

$$reverse(append(X, Y)) = append(reverse(Y), reverse(X))$$

assuming you have the associativity of append available as a rewrite rule:

$$append(A, append(B, C)) = append(append(A, B), C)$$

Exercise 3

Apply Boyer and Moore's techniques to prove that

$$length(append(X, Y)) = plus(length(X), length(Y))$$