Global Types with Internal Delegation and Connecting Communications

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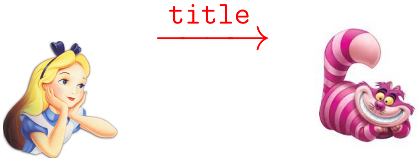
Alice Cat Bank Example

Alice Cat Bank Example



Alice Cat Bank Example

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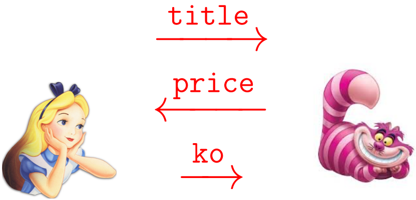


Alice Cat Bank Example

Alice Cat Bank Example

Alice Cat Bank Example

title $\longrightarrow$

price $\longleftarrow$

ok $\longrightarrow$

card $\longrightarrow$

date $\longleftarrow$

Alice Cat Bank Example

$\xrightarrow{\texttt{title}}$

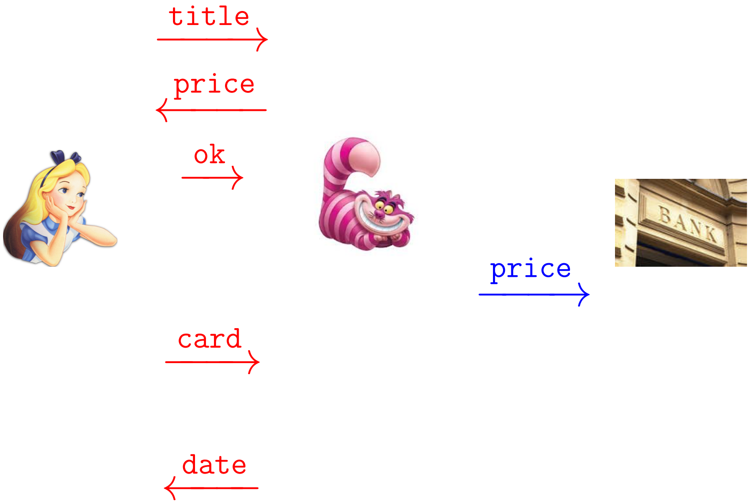
$\xleftarrow{\texttt{price}}$

$\xrightarrow{\texttt{ok}}$

$\xrightarrow{\texttt{card}}$

$\xleftarrow{\texttt{date}}$

Alice Cat Bank Example



Alice Cat Bank Example

$\xrightarrow{\texttt{title}}$

$\xleftarrow{\texttt{price}}$

$\xrightarrow{\texttt{ok}}$

$\xrightarrow{\texttt{card}}$

$\xleftarrow{\texttt{date}}$

$\xrightarrow{\texttt{price}}$

$\xrightarrow{\texttt{channel}}$

Alice Cat Bank Example

$\xrightarrow{\texttt{title}}$

$\xleftarrow{\texttt{price}}$

$\xrightarrow{\texttt{ok}}$

$\xrightarrow{\texttt{price}}$

$\xrightarrow{\texttt{channel}}$

$\xrightarrow{\texttt{card}}$

$\xleftarrow{\texttt{channel}}$

$\xleftarrow{\texttt{date}}$

Two Global Types

G_{ac}

$$\begin{array}{l} \mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C}; \\ \mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A}; \\ \quad (\ (\ \mathtt{A} \xrightarrow{\mathtt{ok}} \mathtt{C}; \\ \qquad \mathtt{A} \xrightarrow{\mathtt{card}} \mathtt{C}; \\ \qquad \mathtt{C} \xrightarrow{\mathtt{date}} \mathtt{A}; \mathsf{End}\ ) \\ \qquad \boxplus \\ \qquad \mathtt{A} \xrightarrow{\mathtt{ko}} \mathtt{C}; \mathsf{End} \\ \quad) \end{array}$$

G_{cb}

$$\begin{array}{l} \mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{B}; \\ \mathtt{C} \xrightarrow{\mathsf{T}} \mathtt{B}; \\ \mathtt{B} \xrightarrow{\mathsf{T}'} \mathtt{C}; \mathsf{End} \end{array}$$

$\mathsf{T} = \mathtt{A}?\,\mathtt{card}\,;\mathsf{T}'$ $\qquad$ $\mathsf{T}' = \mathtt{A}!\,\mathtt{date}\,;\mathsf{End}$

One Global Type

$$\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C};$$

One Global Type

$$\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C};$$
$$\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A};$$

One Global Type

$$
\begin{array}{l}
\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C}; \\
\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A}; \\
\qquad\qquad (\quad \mathtt{A} \xrightarrow{\mathtt{ok}} \mathtt{C};
\end{array}
$$

One Global Type

$$
\begin{aligned}
&\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C};\\
&\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A};\\
&\qquad\quad (\ \ \mathtt{A} \xrightarrow{\mathtt{ok}} \mathtt{C};\\
&\qquad\qquad\ \mathtt{C} \xrightarrow{\overset{\mathtt{price}}{\leftrightarrow}} \mathtt{B};\ \ \textit{connecting communication}
\end{aligned}
$$

One Global Type

$$
\begin{array}{l}
\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C};\\
\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A};\\
\qquad (\quad \mathtt{A} \xrightarrow{\mathtt{ok}} \mathtt{C};\\
\qquad\quad\;\; \mathtt{C} \xrightarrow{\overset{\mathtt{price}}{\leftrightarrow}} \mathtt{B};\\
\qquad\quad\;\; \mathtt{C}\circ\langle\!\langle\bullet\mathtt{B};\ \textit{forward delegation}
\end{array}
$$

One Global Type

$$
\begin{array}{l}
\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C};\\
\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A};\\
\qquad (\quad \mathtt{A} \xrightarrow{\mathtt{ok}} \mathtt{C};\\
\qquad\quad\;\; \mathtt{C} \xrightarrow{\overset{\mathtt{price}}{\leftrightarrow}} \mathtt{B};\\
\qquad\quad\;\; \mathtt{C}\circ\langle\!\langle\bullet\mathtt{B};\\
\qquad\quad\;\; \mathtt{A} \xrightarrow{\mathtt{card}} \mathtt{C};
\end{array}
$$

One Global Type

$$\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C};$$
$$\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A};$$
$$(\quad \mathtt{A} \xrightarrow{\mathtt{ok}} \mathtt{C};$$
$$\mathtt{C} \xrightarrow{\overset{\mathtt{price}}{\leftrightarrow}} \mathtt{B};$$
$$\mathtt{C}\circ\langle\!\langle\bullet\mathtt{B};$$
$$\mathtt{A} \xrightarrow{\mathtt{card}} \mathtt{C};$$
$$\mathtt{B}\bullet\rangle\!\rangle\circ\mathtt{C};$$ *backward delegation*

One Global Type

$$
\begin{aligned}
&\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C};\\
&\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A};\\
&\quad(\ \mathtt{A} \xrightarrow{\mathtt{ok}} \mathtt{C};\\
&\qquad \mathtt{C} \xrightarrow{\overset{\mathtt{price}}{\leftrightarrow}} \mathtt{B};\\
&\qquad {\color{red}\mathtt{C}\circ\langle\!\langle\bullet\mathtt{B}};\\
&\qquad \mathtt{A} \xrightarrow{\mathtt{card}} \mathtt{C};\\
&\qquad {\color{blue}\mathtt{B}\bullet\rangle\!\rangle\circ\mathtt{C}};\\
&\qquad \mathtt{C} \xrightarrow{\mathtt{date}} \mathtt{A};\ \mathsf{End}
\end{aligned}
$$

One Global Type

$$
\begin{array}{l}
\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C};\\
\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A};\\
\qquad (\quad \mathtt{A} \xrightarrow{\mathtt{ok}} \mathtt{C};\\
\qquad\quad\; \mathtt{C} \xrightarrow{\overset{\mathtt{price}}{\leftrightarrow}} \mathtt{B};\\
\qquad\quad\; {\color{red}\mathtt{C}\circ\langle\!\langle\bullet\mathtt{B}};\\
\qquad\quad\; \mathtt{A} \xrightarrow{\mathtt{card}} \mathtt{C};\\
\qquad\quad\; {\color{blue}\mathtt{B}\bullet\rangle\!\rangle\circ\mathtt{C}};\\
\qquad\quad\; \mathtt{C} \xrightarrow{\mathtt{date}} \mathtt{A}; \mathsf{End}\\
\qquad\quad\; \boxplus
\end{array}
$$

One Global Type

$$
\begin{aligned}
&\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C};\\
&\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A};\\
&\qquad\Big(\ \mathtt{A} \xrightarrow{\mathtt{ok}} \mathtt{C};\\
&\qquad\quad \mathtt{C} \xrightarrow{\overset{\mathtt{price}}{\leftrightarrow}} \mathtt{B};\\
&\qquad\quad \mathtt{C}\circ\langle\!\langle\bullet\mathtt{B};\\
&\qquad\quad \mathtt{A} \xrightarrow{\mathtt{card}} \mathtt{C};\\
&\qquad\quad \mathtt{B}\bullet\rangle\!\rangle\circ\mathtt{C};\\
&\qquad\quad \mathtt{C} \xrightarrow{\mathtt{date}} \mathtt{A};\mathsf{End}\\
&\qquad\quad \boxplus\\
&\qquad\quad \mathtt{A} \xrightarrow{\mathtt{ko}} \mathtt{C};\mathsf{End}\ \Big)
\end{aligned}
$$

Start with **Forward delegation**

$$\mathtt{C}\circ\langle\!\langle\bullet\mathtt{B}$$

End $\qquad$ $\circ\langle\!\langle\bullet\mathtt{B}$ $\qquad$ $\mathtt{C}\circ\langle\!\langle\bullet$

Terminology (active/passive):

- **active forward delegation** $\circ\langle\!\langle\bullet\mathtt{B}$
- **passive forward delegation** $\mathtt{C}\circ\langle\!\langle\bullet$.

Message sent to Cat goes directly to Bank

$$\mathtt{C}\circ\langle\!\langle\bullet\mathtt{B};$$
$$\mathtt{A} \xrightarrow{\mathtt{card}} \mathtt{C}$$

$\mathtt{C}!\,\mathtt{card}\,;$

$\circ\langle\!\langle\bullet\mathtt{B}$

$\mathtt{C}\circ\langle\!\langle\bullet;$
$\mathtt{A}?\,\mathtt{card}$

Trust assumption: Cat does not have authority to handle card.

End with **backward delegation**

$$\mathtt{C}\circ\langle\!\langle\bullet\mathtt{B};$$
$$\mathtt{A}\xrightarrow{\mathtt{card}}\mathtt{C};$$
$$\mathtt{B}\bullet\rangle\!\rangle\circ\mathtt{C};$$

$\mathtt{C}!\,\mathtt{card}\,;$

$\circ\langle\!\langle\bullet\mathtt{B};\mathtt{B}\bullet\rangle\!\rangle\circ$

$\mathtt{C}\circ\langle\!\langle\bullet;$
$\mathtt{A}?\,\mathtt{card}\,;$
$\bullet\rangle\!\rangle\circ\mathtt{C}$

Terminology (active/passive):

- **active backward delegation** $\bullet\rangle\!\rangle\circ\mathtt{C}$
- **passive backward delegation** $\mathtt{B}\bullet\rangle\!\rangle\circ$.

Processes

Λ ranges over λ and $\overset{\lambda}{\leftrightarrow}$

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$$P \quad ::= \quad \Sigma_{i \in I} \mathsf{p}_i?\Lambda_i; P_i \quad \textit{external choices of inputs}$$

Processes

Λ ranges over λ and $\overset{\lambda}{\leftrightarrow}$

$$P \quad ::= \quad \Sigma_{i \in I} \mathsf{p}_i?\Lambda_i; P_i \quad | \quad \bigoplus_{i \in I} \mathsf{p}_i!\Lambda_i; P_i \ \textit{internal choices of outputs}$$

Processes

Λ ranges over λ and $\overset{\lambda}{\leftrightarrow}$

$$
\begin{array}{rcl}
P & ::= & \Sigma_{i \in I} \mathsf{p}_i?\Lambda_i; P_i \quad | \quad \bigoplus_{i \in I} \mathsf{p}_i!\Lambda_i; P_i \\
 & | & \mathsf{p} \circ \langle\!\langle \bullet; P \ \textit{forward delegation with principal}
\end{array}
$$

Processes

Λ ranges over λ and $\overset{\lambda}{\leftrightarrow}$

$$
\begin{array}{rcl}
P & ::= & \Sigma_{i\in I}\mathsf{p}_i?\Lambda_i; P_i \quad | \quad \bigoplus_{i\in I}\, \mathsf{p}_i!\Lambda_i; P_i \\
 & | & \mathsf{p}\circ\langle\!\langle\bullet; P \quad | \quad \circ\langle\!\langle\bullet\mathsf{p}; P \ \textit{forward delegation with deputy}
\end{array}
$$

Processes

Λ ranges over λ and $\overset{\lambda}{\leftrightarrow}$

$$
\begin{array}{rcl}
P & ::= & \Sigma_{i\in I}\mathsf{p}_i?\Lambda_i;P_i \quad | \quad \bigoplus_{i\in I}\mathsf{p}_i!\Lambda_i;P_i \\
& | & \mathsf{p}\circ\langle\!\langle\bullet;P \quad | \quad \circ\langle\!\langle\bullet\mathsf{p};P \\
& | & \bullet\rangle\!\rangle\circ\mathsf{q};P \ \textit{backward delegation with principal}
\end{array}
$$

Processes

Λ ranges over λ and $\overset{\lambda}{\leftrightarrow}$

$$
\begin{array}{llllll}
P & ::= & \Sigma_{i\in I}\mathsf{p}_i?\Lambda_i;P_i & \mid & \bigoplus_{i\in I}\mathsf{p}_i!\Lambda_i;P_i & \\
& \mid & \mathsf{p}\circ\langle\!\langle\bullet;P & \mid & \circ\langle\!\langle\bullet\mathsf{p};P & \\
& \mid & \bullet\rangle\!\rangle\circ\mathsf{q};P & \mid & \mathsf{q}\bullet\rangle\!\rangle\circ;P & \textit{backward delegation with deputy}
\end{array}
$$

Processes

Λ ranges over λ and $\overset{\lambda}{\leftrightarrow}$

$$
\begin{array}{rcl}
P & ::= & \Sigma_{i\in I}\mathsf{p}_i?\Lambda_i;P_i \quad | \quad \bigoplus_{i\in I}\mathsf{p}_i!\Lambda_i;P_i \\
& | & \mathsf{p}\circ\langle\!\langle\bullet;P \quad | \quad \circ\langle\!\langle\bullet\mathsf{p};P \\
& | & \bullet\rangle\!\rangle\circ\mathsf{q};P \quad | \quad \mathsf{q}\bullet\rangle\!\rangle\circ;P \\
& | & \mu X.P \quad | \quad X \quad | \quad \mathbf{0}
\end{array}
$$

Processes

Λ ranges over λ and $\overset{\lambda}{\leftrightarrow}$

$$
\begin{array}{rcl}
P & ::= & \Sigma_{i\in I}\mathsf{p}_i?\Lambda_i; P_i \quad | \quad \bigoplus_{i\in I}\mathsf{p}_i!\Lambda_i; P_i \\
 & | & \mathsf{p}\circ\langle\!\langle\bullet; P \quad | \quad \circ\langle\!\langle\bullet\mathsf{p}; P \\
 & | & \bullet\rangle\!\rangle\circ\mathsf{q}; P \quad | \quad \mathsf{q}\bullet\rangle\!\rangle\circ; P \\
 & | & \mu X.P \quad | \quad X \quad | \quad \mathbf{0}
\end{array}
$$

internal and external choices must not be ambiguous

Processes

Λ ranges over λ and $\overset{\lambda}{\leftrightarrow}$

$$
\begin{array}{rcl}
P & ::= & \Sigma_{i\in I}\mathsf{p}_i?\Lambda_i;P_i \quad \mid \quad \bigoplus_{i\in I}\mathsf{p}_i!\Lambda_i;P_i \\
 & \mid & \mathsf{p}\circ\langle\!\langle\bullet;P \quad \mid \quad \circ\langle\!\langle\bullet\mathsf{p};P \\
 & \mid & \bullet\rangle\!\rangle\circ\mathsf{q};P \quad \mid \quad \mathsf{q}\bullet\rangle\!\rangle\circ;P \\
 & \mid & \mu X.P \quad \mid \quad X \quad \mid \quad \mathbf{0}
\end{array}
$$

internal and external choices must not be ambiguous

Processes

Λ ranges over λ and $\overset{\lambda}{\leftrightarrow}$

$$
\begin{array}{rcl}
P & ::= & \Sigma_{i \in I} \mathsf{p}_i ? \Lambda_i ; P_i \quad | \quad \bigoplus_{i \in I} \mathsf{p}_i ! \Lambda_i ; P_i \\
& | & \mathsf{p} \circ\langle\!\langle \bullet ; P \quad | \quad \circ\langle\!\langle \bullet \mathsf{p} ; P \\
& | & \bullet\rangle\!\rangle\circ \mathsf{q} ; P \quad | \quad \mathsf{q}\bullet\rangle\!\rangle\circ ; P \\
& | & \mu X . P \quad | \quad X \quad | \quad \mathbf{0}
\end{array}
$$

internal and external choices must not be ambiguous

$\mathtt{A?title;A!price;(A?ok;B!}\overset{\mathtt{price}}{\leftrightarrow}\mathtt{;}\circ\langle\!\langle\bullet\mathsf{B};\mathsf{B}\bullet\rangle\!\rangle\circ\mathtt{;A!date}+\mathtt{A?ko)}$

Networks

$$\mathtt{A}[\![\, \mathtt{C}!\,\mathtt{card}\,;\mathtt{C}?\,\mathtt{date}\ ]\!] \parallel \mathtt{C}[\![\, {\color{red}\circ\langle\!\langle \bullet \mathtt{B}};{\color{blue}\mathtt{B}\bullet\rangle\!\rangle\circ};\mathtt{A}!\,\mathtt{date}\ ]\!] \parallel$$
$$\mathtt{B}[\![\, {\color{red}\mathtt{C}\circ\langle\!\langle\bullet};\mathtt{A}?\,\mathtt{card}\,;{\color{blue}\bullet\rangle\!\rangle\circ\mathtt{C}}\,]\!]$$

Networks

$$\mathtt{A}[\![\, \mathtt{C}!\,\mathtt{card}\,;\mathtt{C}?\,\mathtt{date}\,]\!] \parallel \mathtt{C}[\![\, \circ\langle\!\langle \bullet\mathtt{B};\, \mathtt{B}\bullet\rangle\!\rangle\circ;\, \mathtt{A}!\,\mathtt{date}\,]\!] \parallel$$
$$\mathtt{B}[\![\, \mathtt{C}\circ\langle\!\langle\bullet;\, \mathtt{A}?\,\mathtt{card}\,;\,\bullet\rangle\!\rangle\circ\mathtt{C}\,]\!]$$
$$\Downarrow$$
$$\mathtt{A}[\![\, \mathtt{C}!\,\mathtt{card}\,;\mathtt{C}?\,\mathtt{date}\,]\!] \parallel \overset{*}{\mathtt{C}}[\![\, \mathtt{B}\bullet\rangle\!\rangle\circ;\, \mathtt{A}!\,\mathtt{date}\,]\!] \parallel \mathtt{C}[\![\, \mathtt{A}?\,\mathtt{card}\,;\,\bullet\rangle\!\rangle\circ\mathtt{C}\,]\!]$$

Networks

$$\mathtt{A}[\![\, \mathtt{C}!\,\mathtt{card}\,;\mathtt{C}?\,\mathtt{date}\,]\!] \parallel \mathtt{C}[\![\, \circ\langle\!\langle \bullet \mathtt{B};\, \mathtt{B}\bullet \rangle\!\rangle \circ;\, \mathtt{A}!\,\mathtt{date}\,]\!] \parallel$$
$$\mathtt{B}[\![\, \mathtt{C}\circ\langle\!\langle \bullet;\, \mathtt{A}?\,\mathtt{card}\,;\, \bullet\rangle\!\rangle \circ\mathtt{C}\,]\!]$$
$$\Downarrow$$
$$\mathtt{A}[\![\, \mathtt{C}!\,\mathtt{card}\,;\mathtt{C}?\,\mathtt{date}\,]\!] \parallel \overset{*}{\mathtt{C}}[\![\, \mathtt{B}\bullet \rangle\!\rangle \circ;\, \mathtt{A}!\,\mathtt{date}\,]\!] \parallel \mathtt{C}[\![\, \mathtt{A}?\,\mathtt{card}\,;\, \bullet\rangle\!\rangle \circ\mathtt{C}\,]\!]$$
$$\Downarrow$$
$$\mathtt{A}[\![\, \mathtt{C}?\,\mathtt{date}\,]\!] \parallel \overset{*}{\mathtt{C}}[\![\, \mathtt{B}\bullet \rangle\!\rangle \circ;\, \mathtt{A}!\,\mathtt{date}\,]\!] \parallel \mathtt{C}[\![\, \bullet\rangle\!\rangle \circ\mathtt{C}\,]\!]$$

Networks

$$\mathtt{A}[\![\, \mathtt{C}!\,\mathtt{card}\,;\mathtt{C}?\,\mathtt{date}\,]\!] \parallel \mathtt{C}[\![\, \circ\langle\!\langle \bullet\mathtt{B};\, \mathtt{B}\bullet\rangle\!\rangle\circ;\, \mathtt{A}!\,\mathtt{date}\,]\!] \parallel \mathtt{B}[\![\, \mathtt{C}\circ\langle\!\langle\bullet;\, \mathtt{A}?\,\mathtt{card}\,;\,\bullet\rangle\!\rangle\circ\mathtt{C}\,]\!]$$
$$\Downarrow$$
$$\mathtt{A}[\![\, \mathtt{C}!\,\mathtt{card}\,;\mathtt{C}?\,\mathtt{date}\,]\!] \parallel \overset{*}{\mathtt{C}}[\![\, \mathtt{B}\bullet\rangle\!\rangle\circ;\, \mathtt{A}!\,\mathtt{date}\,]\!] \parallel \mathtt{C}[\![\, \mathtt{A}?\,\mathtt{card}\,;\,\bullet\rangle\!\rangle\circ\mathtt{C}\,]\!]$$
$$\Downarrow$$
$$\mathtt{A}[\![\, \mathtt{C}?\,\mathtt{date}\,]\!] \parallel \overset{*}{\mathtt{C}}[\![\, \mathtt{B}\bullet\rangle\!\rangle\circ;\, \mathtt{A}!\,\mathtt{date}\,]\!] \parallel \mathtt{C}[\![\, \bullet\rangle\!\rangle\circ\mathtt{C}\,]\!]$$
$$\Downarrow$$
$$\mathtt{A}[\![\, \mathtt{C}?\,\mathtt{date}\,]\!] \parallel \mathtt{C}[\![\, \mathtt{A}!\,\mathtt{date}\,]\!] \parallel \mathtt{B}[\![\, \mathbf{0}\,]\!]$$

Networks

$$\mathtt{A}[\![\, \mathtt{C}!\,\mathtt{card}\,;\mathtt{C}?\,\mathtt{date}\,]\!] \parallel \mathtt{C}[\![\, \circ\langle\!\langle \bullet \mathrm{B}; \mathrm{B}\bullet \rangle\!\rangle\circ; \mathtt{A}!\,\mathtt{date}\,]\!] \parallel$$
$$\mathtt{B}[\![\, \mathrm{C}\circ\langle\!\langle \bullet; \mathtt{A}?\,\mathtt{card}\,; \bullet\rangle\!\rangle\circ\mathrm{C}\,]\!]$$
$$\Downarrow$$
$$\mathtt{A}[\![\, \mathtt{C}!\,\mathtt{card}\,;\mathtt{C}?\,\mathtt{date}\,]\!] \parallel \overset{*}{\mathtt{C}}[\![\, \mathrm{B}\bullet\rangle\!\rangle\circ; \mathtt{A}!\,\mathtt{date}\,]\!] \parallel \mathtt{C}[\![\, \mathtt{A}?\,\mathtt{card}\,; \bullet\rangle\!\rangle\circ\mathrm{C}\,]\!]$$
$$\Downarrow$$
$$\mathtt{A}[\![\, \mathtt{C}?\,\mathtt{date}\,]\!] \parallel \overset{*}{\mathtt{C}}[\![\, \mathrm{B}\bullet\rangle\!\rangle\circ; \mathtt{A}!\,\mathtt{date}\,]\!] \parallel \mathtt{C}[\![\, \bullet\rangle\!\rangle\circ\mathrm{C}\,]\!]$$
$$\Downarrow$$
$$\mathtt{A}[\![\, \mathtt{C}?\,\mathtt{date}\,]\!] \parallel \mathtt{C}[\![\, \mathtt{A}!\,\mathtt{date}\,]\!] \parallel \mathtt{B}[\![\, \mathbf{0}\,]\!]$$

$$\mathbb{N} \quad ::= \quad \mathsf{p}[\![\, P \,]\!] \quad | \quad \overset{*}{\mathsf{p}}[\![\, P \,]\!] \quad | \quad \mathbb{N} \parallel \mathbb{N}$$

Operational Semantics

$$\Sigma_{i \in I} \mathsf{p}_i ? \Lambda_i ; P_i \xrightarrow{\mathsf{p}_j ? \Lambda_j} P_j \quad j \in I \qquad [\textsc{ExtCh}]$$

Operational Semantics

$$\Sigma_{i\in I}\mathsf{p}_i?\Lambda_i; P_i \xrightarrow{\mathsf{p}_j?\Lambda_j} P_j \quad j \in I \qquad [\textsc{ExtCh}]$$

$$\bigoplus_{i\in I}\mathsf{p}_i!\Lambda_i; P_i \xrightarrow{\mathsf{p}_j!\Lambda_j} P_j \quad j \in I \qquad [\textsc{IntCh}]$$

Operational Semantics

$$\frac{P \xrightarrow{\mathsf{q}!\Lambda} P' \quad Q \xrightarrow{\mathsf{p}?\Lambda} Q'}{\mathsf{p}[\![\, P \,]\!] \parallel \mathsf{q}[\![\, Q \,]\!] \xrightarrow{\mathsf{p}\Lambda\mathsf{q}} \mathsf{p}[\![\, P' \,]\!] \parallel \mathsf{q}[\![\, Q' \,]\!]}\ [\textsc{Com}]$$

Operational Semantics

$$\frac{P \xrightarrow{\mathsf{q}!\Lambda} P' \quad Q \xrightarrow{\mathsf{p}?\Lambda} Q'}{\mathsf{p}[\![\, P \,]\!] \parallel \mathsf{q}[\![\, Q \,]\!] \xrightarrow{\mathsf{p}\Lambda\mathsf{q}} \mathsf{p}[\![\, P' \,]\!] \parallel \mathsf{q}[\![\, Q' \,]\!]} \; [\textsc{Com}]$$

$$\mathsf{p}[\![\, \circ\langle\!\langle \bullet \mathsf{q}; P \,]\!] \parallel \mathsf{q}[\![\, \mathsf{p} \circ\langle\!\langle \bullet; Q \,]\!] \xrightarrow{\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}} \overset{*}{\mathsf{p}}[\![\, P \,]\!] \parallel \mathsf{p}[\![\, Q \,]\!] \quad [\textsc{BDel}]$$

Operational Semantics

$$\frac{P \xrightarrow{\mathsf{q}!\Lambda} P' \quad Q \xrightarrow{\mathsf{p}?\Lambda} Q'}{\mathsf{p}[\![\, P \,]\!] \parallel \mathsf{q}[\![\, Q \,]\!] \xrightarrow{\mathsf{p}\Lambda\mathsf{q}} \mathsf{p}[\![\, P' \,]\!] \parallel \mathsf{q}[\![\, Q' \,]\!]} \ [\textsc{Com}]$$

$$\mathsf{p}[\![\, \circ\langle\!\langle \bullet \mathsf{q}; P \,]\!] \parallel \mathsf{q}[\![\, \mathsf{p} \circ\langle\!\langle \bullet; Q \,]\!] \xrightarrow{\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}} \overset{*}{\mathsf{p}}[\![\, P \,]\!] \parallel \mathsf{p}[\![\, Q \,]\!] \quad [\textsc{BDel}]$$

$$\overset{*}{\mathsf{p}}[\![\, \mathsf{q} \bullet\rangle\!\rangle\circ; P \,]\!] \parallel \mathsf{p}[\![\, \bullet\rangle\!\rangle\circ\mathsf{p}; Q \,]\!] \xrightarrow{\mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}} \mathsf{p}[\![\, P \,]\!] \parallel \mathsf{q}[\![\, Q \,]\!] \quad [\textsc{EDel}]$$

Operational Semantics

$$\frac{P \xrightarrow{\mathsf{q}!\Lambda} P' \quad Q \xrightarrow{\mathsf{p}?\Lambda} Q'}{\mathsf{p}[\![\, P \,]\!] \parallel \mathsf{q}[\![\, Q \,]\!] \xrightarrow{\mathsf{p}\Lambda\mathsf{q}} \mathsf{p}[\![\, P' \,]\!] \parallel \mathsf{q}[\![\, Q' \,]\!]} \; [\textsc{Com}]$$

$$\mathsf{p}[\![\, \circ\langle\!\langle \bullet \mathsf{q}; P \,]\!] \parallel \mathsf{q}[\![\, \mathsf{p} \circ\langle\!\langle \bullet; Q \,]\!] \xrightarrow{\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}} \overset{*}{\mathsf{p}}[\![\, P \,]\!] \parallel \mathsf{p}[\![\, Q \,]\!] \quad [\textsc{BDel}]$$

$$\overset{*}{\mathsf{p}}[\![\, \mathsf{q} \bullet\rangle\!\rangle\circ; P \,]\!] \parallel \mathsf{p}[\![\, \bullet\rangle\!\rangle\circ\mathsf{p}; Q \,]\!] \xrightarrow{\mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}} \mathsf{p}[\![\, P \,]\!] \parallel \mathsf{q}[\![\, Q \,]\!] \quad [\textsc{EDel}]$$

$$\frac{\mathbb{N} \xrightarrow{\phi} \mathbb{N}'}{\mathbb{N} \parallel \mathbb{N}'' \xrightarrow{\phi} \mathbb{N}' \parallel \mathbb{N}''} \; [\textsc{Ct}]$$

ϕ ranges over $\mathsf{p}\Lambda\mathsf{q}$, $\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}$, $\mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}$

Partial Order on Processes

a process offering more inputs and less outputs is better

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[Sub-In]

$$\frac{\forall i \in I : P_i \leq Q_i}{\Sigma_{i \in I \cup J} \mathsf{p}_i?\Lambda_i; P_i \leq \Sigma_{i \in I} \mathsf{p}_i?\Lambda_i; Q_i}$$

[Sub-Out]

$$\frac{\forall i \in I : P_i \leq Q_i}{\oplus_{i \in I} \mathsf{p}_i!\Lambda_i; P_i \leq \oplus_{i \in I \cup J} \mathsf{p}_i!\Lambda_i; Q_i}$$

Partial Order on Processes

[Sub-In]

$$\frac{\forall i \in I : P_i \leq Q_i}{\Sigma_{i \in I \cup J} \mathsf{p}_i?\Lambda_i; P_i \leq \Sigma_{i \in I} \mathsf{p}_i?\Lambda_i; Q_i}$$

[Sub-Out]

$$\frac{\forall i \in I : P_i \leq Q_i}{\oplus_{i \in I} \mathsf{p}_i!\Lambda_i; P_i \leq \oplus_{i \in I \cup J} \mathsf{p}_i!\Lambda_i; Q_i}$$

connecting communications are better than $\mathbf{0}$

Partial Order on Processes

[SUB-IN]
$$\frac{\forall i \in I : P_i \leq Q_i}{\Sigma_{i \in I \cup J} \mathsf{p}_i?\Lambda_i; P_i \leq \Sigma_{i \in I} \mathsf{p}_i?\Lambda_i; Q_i}$$

[SUB-OUT]
$$\frac{\forall i \in I : P_i \leq Q_i}{\oplus_{i \in I} \mathsf{p}_i!\Lambda_i; P_i \leq \oplus_{i \in I \cup J} \mathsf{p}_i!\Lambda_i; Q_i}$$

connecting communications are better than $\mathbf{0}$

[SUB-IN-SKIP]
$$\frac{}{\Sigma_{i \in I} \mathsf{p}_i?\overset{\lambda_i}{\leftrightarrow}; P_i \leq \mathbf{0}}$$

Partial Order on Processes

$$[\textsc{Sub-In}]\ \dfrac{\forall i \in I : P_i \leq Q_i}{\Sigma_{i \in I \cup J} \mathsf{p}_i?\Lambda_i; P_i \leq \Sigma_{i \in I} \mathsf{p}_i?\Lambda_i; Q_i}$$

$$[\textsc{Sub-Out}]\ \dfrac{\forall i \in I : P_i \leq Q_i}{\bigoplus_{i \in I} \mathsf{p}_i!\Lambda_i; P_i \leq \bigoplus_{i \in I \cup J} \mathsf{p}_i!\Lambda_i; Q_i}$$

$$[\textsc{Sub-In-Skip}]\ \dfrac{}{\Sigma_{i \in I} \mathsf{p}_i?\overset{\lambda_i}{\leftrightarrow}; P_i \leq \mathbf{0}}$$

δ *ranges over* $\circ\langle\!\langle \bullet\ \bullet \rangle\!\rangle\circ$

Partial Order on Processes

[Sub-In]

$$\frac{\forall i \in I : P_i \leq Q_i}{\Sigma_{i \in I \cup J} \mathrm{p}_i ? \Lambda_i ; P_i \leq \Sigma_{i \in I} \mathrm{p}_i ? \Lambda_i ; Q_i}$$

[Sub-Out]

$$\frac{\forall i \in I : P_i \leq Q_i}{\oplus_{i \in I} \mathrm{p}_i ! \Lambda_i ; P_i \leq \oplus_{i \in I \cup J} \mathrm{p}_i ! \Lambda_i ; Q_i}$$

[Sub-In-Skip]

$$\frac{}{\Sigma_{i \in I} \mathrm{p}_i ? \overset{\lambda_i}{\leftrightarrow} ; P_i \leq \mathbf{0}}$$

δ *ranges over* $\circ\langle\!\langle \bullet \ \bullet \rangle\!\rangle\circ$

[Sub-Del]

$$\frac{P \leq Q}{\delta ; P \leq \delta ; Q}$$

Partial Order on Processes

[Sub-In]

$$\frac{\forall i \in I : P_i \leq Q_i}{\Sigma_{i \in I \cup J} \mathsf{p}_i?\Lambda_i; P_i \leq \Sigma_{i \in I} \mathsf{p}_i?\Lambda_i; Q_i}$$

[Sub-Out]

$$\frac{\forall i \in I : P_i \leq Q_i}{\oplus_{i \in I} \mathsf{p}_i!\Lambda_i; P_i \leq \oplus_{i \in I \cup J} \mathsf{p}_i!\Lambda_i; Q_i}$$

[Sub-In-Skip]

$$\frac{}{\Sigma_{i \in I} \mathsf{p}_i?\overset{\lambda_i}{\leftrightarrow}; P_i \leq \mathbf{0}}$$

[Sub-Del]

$$\frac{P \leq Q}{\delta; P \leq \delta; Q}$$

[Sub-**0**]

$$\mathbf{0} \leq \mathbf{0}$$

Global types

$$
\begin{array}{l}
\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C};\\
\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A};\\
\qquad\quad \Big(\quad \mathtt{A} \xrightarrow{\mathtt{ok}} \mathtt{C};\\
\qquad\qquad\quad \mathtt{C} \xrightarrow{\overset{\mathtt{price}}{\leftrightarrow}} \mathtt{B};\\
\qquad\qquad\quad \mathtt{C}\circ\langle\!\langle\bullet\mathtt{B};\\
\qquad\qquad\quad \mathtt{A} \xrightarrow{\mathtt{card}} \mathtt{C};\\
\qquad\qquad\quad \mathtt{B}\bullet\rangle\!\rangle\circ\mathtt{C};\\
\qquad\qquad\quad \mathtt{C} \xrightarrow{\mathtt{date}} \mathtt{A};\mathsf{End}\\
\qquad\qquad\quad \boxplus\\
\qquad\qquad\quad \mathtt{A} \xrightarrow{\mathtt{ko}} \mathtt{C};\mathsf{End}\quad \Big)
\end{array}
$$

Global types

$$
\begin{array}{l}
\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C};\\
\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A};\\
\qquad (\quad \mathtt{A} \xrightarrow{\mathtt{ok}} \mathtt{C};\\
\qquad\quad\ \mathtt{C} \xrightarrow{\overset{\mathtt{price}}{\leftrightarrow}} \mathtt{B};\\
\qquad\quad\ \mathtt{C}\circ\langle\!\langle\bullet\mathtt{B};\\
\qquad\quad\ \mathtt{A} \xrightarrow{\mathtt{card}} \mathtt{C};\\
\qquad\quad\ \mathtt{B}\bullet\rangle\!\rangle\circ\mathtt{C};\\
\qquad\quad\ \mathtt{C} \xrightarrow{\mathtt{date}} \mathtt{A}; \mathsf{End}\\
\qquad\quad\ \boxplus\\
\qquad\quad\ \mathtt{A} \xrightarrow{\mathtt{ko}} \mathtt{C}; \mathsf{End} \quad)
\end{array}
$$

$$
\begin{array}{rcl}
\mathsf{G} & ::= & \boxplus_{i\in I}\, \mathsf{p}\Lambda_i \mathsf{q}_i; \mathsf{G}_i\\
& | & \mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}; \mathsf{G} \quad | \quad \mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}; \mathsf{G}\\
& | & \mu\mathbf{t}.\mathsf{G} \quad | \quad \mathbf{t} \quad | \quad \mathsf{End}
\end{array}
$$

Global types

$$
\begin{array}{l}
\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C};\\
\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A};\\
\qquad (\ \ \mathtt{A} \xrightarrow{\mathtt{ok}} \mathtt{C};\\
\qquad\quad\ \ \mathtt{C} \xrightarrow{\overset{\mathtt{price}}{\leftrightarrow}} \mathtt{B};\\
\qquad\quad\ \ {\color{red}\mathtt{C}\circ\langle\!\langle\bullet\mathtt{B}};\\
\qquad\quad\ \ \mathtt{A} \xrightarrow{\mathtt{card}} \mathtt{C};\\
\qquad\quad\ \ {\color{blue}\mathtt{B}\bullet\rangle\!\rangle\circ\mathtt{C}};\\
\qquad\quad\ \ \mathtt{C} \xrightarrow{\mathtt{date}} \mathtt{A};\mathsf{End}\\
\qquad\quad\ \ \boxplus\\
\qquad\quad\ \ \mathtt{A} \xrightarrow{\mathtt{ko}} \mathtt{C};\mathsf{End}\ \ )
\end{array}
$$

$$
\begin{array}{lcl}
\mathsf{G} & ::= & \boxplus_{i\in I}\, \mathsf{p}\Lambda_i\mathsf{q}_i; \mathsf{G}_i\\
& | & {\color{red}\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}}; \mathsf{G} \ \ |\ \ {\color{blue}\mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}}; \mathsf{G}\\
& | & \mu\mathbf{t}.\mathsf{G} \ \ |\ \ \mathbf{t} \ \ |\ \ \mathsf{End}
\end{array}
$$

- no ambiguity of choices

Global types

$$
\begin{array}{l}
\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C};\\
\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A};\\
\quad (\quad \mathtt{A} \xrightarrow{\mathtt{ok}} \mathtt{C};\\
\qquad \mathtt{C} \xrightarrow{\overset{\mathtt{price}}{\leftrightarrow}} \mathtt{B};\\
\qquad {\color{red}\mathtt{C}\circ\langle\!\langle\bullet\mathtt{B}};\\
\qquad \mathtt{A} \xrightarrow{\mathtt{card}} \mathtt{C};\\
\qquad {\color{blue}\mathtt{B}\bullet\rangle\!\rangle\circ\mathtt{C}};\\
\qquad \mathtt{C} \xrightarrow{\mathtt{date}} \mathtt{A}; \mathsf{End}\\
\qquad \boxplus\\
\qquad \mathtt{A} \xrightarrow{\mathtt{ko}} \mathtt{C}; \mathsf{End} \quad)
\end{array}
$$

$$
\begin{array}{rcl}
\mathsf{G} & ::= & \boxplus_{i\in I}\, \mathsf{p}\Lambda_i\mathsf{q}_i; \mathsf{G}_i\\
& | & {\color{red}\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}}; \mathsf{G} \quad | \quad {\color{blue}\mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}}; \mathsf{G}\\
& | & \mu\mathbf{t}.\mathsf{G} \quad | \quad \mathbf{t} \quad | \quad \mathsf{End}
\end{array}
$$

- no ambiguity of choices
- each occurrence of $\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}$ is followed by an occurrence of $\mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}$

Global types

$$
\begin{array}{l}
\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C};\\
\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A};\\
\quad (\quad \mathtt{A} \xrightarrow{\mathtt{ok}} \mathtt{C};\\
\qquad \mathtt{C} \xrightarrow{\overset{\mathtt{price}}{\leftrightarrow}} \mathtt{B};\\
\qquad \color{red}{\mathtt{C}\circ\langle\!\langle\bullet\mathtt{B}};\\
\qquad \mathtt{A} \xrightarrow{\mathtt{card}} \mathtt{C};\\
\qquad \color{blue}{\mathtt{B}\bullet\rangle\!\rangle\circ\mathtt{C}};\\
\qquad \mathtt{C} \xrightarrow{\mathtt{date}} \mathtt{A}; \mathsf{End}\\
\qquad \boxplus\\
\qquad \mathtt{A} \xrightarrow{\mathtt{ko}} \mathtt{C}; \mathsf{End} \quad)
\end{array}
$$

$$
\begin{array}{lll}
\mathsf{G} & ::= & \boxplus_{i\in I}\, \mathsf{p}\Lambda_i\mathsf{q}_i; \mathsf{G}_i\\
& \mid & \color{red}{\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}}; \mathsf{G} \;\mid\; \color{blue}{\mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}}; \mathsf{G}\\
& \mid & \mu\mathbf{t}.\mathsf{G} \;\mid\; \mathbf{t} \;\mid\; \mathsf{End}
\end{array}
$$

- no ambiguity of choices
- each occurrence of $\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}$ is followed by an occurrence of $\mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}$
- no atomic interaction involving q occurs between $\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}$ and $\mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}$

Global types

$\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C};$
$\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A};$
$($ $\mathtt{A} \xrightarrow{\mathtt{ok}} \mathtt{C};$
$\mathtt{C} \xrightarrow{\overset{\mathtt{price}}{\leftrightarrow}} \mathtt{B};$
$\mathtt{C}\circ\langle\!\langle\bullet\mathtt{B};$
$\mathtt{A} \xrightarrow{\mathtt{card}} \mathtt{C};$
$\mathtt{B}\bullet\rangle\!\rangle\circ\mathtt{C};$
$\mathtt{C} \xrightarrow{\mathtt{date}} \mathtt{A}; \mathsf{End}$
$\boxplus$
$\mathtt{A} \xrightarrow{\mathtt{ko}} \mathtt{C}; \mathsf{End}$ $)$

$$\begin{array}{rcl} \mathsf{G} & ::= & \boxplus_{i\in I}\, \mathsf{p}\Lambda_i\mathsf{q}_i; \mathsf{G}_i \\ & \mid & \mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}; \mathsf{G} \;\mid\; \mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}; \mathsf{G} \\ & \mid & \mu\mathbf{t}.\mathsf{G} \;\mid\; \mathbf{t} \;\mid\; \mathsf{End} \end{array}$$

- no ambiguity of choices
- each occurrence of $\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}$ is followed by an occurrence of $\mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}$
- no atomic interaction involving q occurs between $\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}$ and $\mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}$
- no choice occurs between $\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}$ and $\mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}$

Global types

$$\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C};$$
$$\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A};$$
$$(\quad \mathtt{A} \xrightarrow{\mathtt{ok}} \mathtt{C};$$
$$\mathtt{C} \xrightarrow{\overset{\mathtt{price}}{\leftrightarrow}} \mathtt{B};$$
$$\mathtt{C}\circ\langle\!\langle\bullet\mathtt{B};$$
$$\mathtt{A} \xrightarrow{\mathtt{card}} \mathtt{C};$$
$$\mathtt{B}\bullet\rangle\!\rangle\circ\mathtt{C};$$
$$\mathtt{C} \xrightarrow{\mathtt{date}} \mathtt{A}; \mathsf{End}$$
$$\boxplus$$
$$\mathtt{A} \xrightarrow{\mathtt{ko}} \mathtt{C}; \mathsf{End} \quad)$$

$$\mathsf{G} \quad ::= \quad \boxplus_{i\in I}\, \mathsf{p}\Lambda_i\mathsf{q}_i; \mathsf{G}_i \;\mid\; \mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}; \mathsf{G} \;\mid\; \mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}; \mathsf{G} \;\mid\; \mu\mathbf{t}.\mathsf{G} \;\mid\; \mathbf{t} \;\mid\; \mathsf{End}$$

- no ambiguity of choices
- each occurrence of $\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}$ is followed by an occurrence of $\mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}$
- no atomic interaction involving q occurs between $\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}$ and $\mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}$
- no choice occurs between $\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}$ and $\mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}$
- no delegation involving p occurs between $\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}$ and $\mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}$

Projection: Example

$$
\begin{array}{l}
\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C};\\
\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A};\\
\qquad\Big(\;\mathtt{A} \xrightarrow{\mathtt{ok}} \mathtt{C};\\
\qquad\quad \mathtt{C} \xrightarrow{\overset{\mathtt{price}}{\leftrightarrow}} \mathtt{B};\\
\qquad\quad \mathtt{C}\circ\langle\!\langle\bullet\mathtt{B};\\
\qquad\quad \mathtt{A} \xrightarrow{\mathtt{card}} \mathtt{C};\\
\qquad\quad \mathtt{B}\bullet\rangle\!\rangle\circ\mathtt{C};\\
\qquad\quad \mathtt{C} \xrightarrow{\mathtt{date}} \mathtt{A};\ \mathsf{End}\\
\qquad\quad \boxplus\\
\qquad\quad \mathtt{A} \xrightarrow{\mathtt{ko}} \mathtt{C};\ \mathsf{End}\quad\Big)
\end{array}
$$

Projection: Example

$$
\begin{array}{l}
\mathtt{A}?\,\mathtt{title};\\
\mathtt{A}!\,\mathtt{price};\\
\qquad (\ \ \mathtt{A}?\,\mathtt{ok};\\
\qquad\quad \mathtt{B}!\,\overset{\mathtt{price}}{\leftrightarrow};\\
\qquad\quad \circ\langle\!\langle\bullet\mathtt{B};\\
\\
\qquad\quad \mathtt{B}\bullet\rangle\!\rangle\circ;\\
\qquad\quad \mathtt{A}!\,\mathtt{date};\\
\qquad\quad +\\
\qquad\quad \mathtt{A}?\,\mathtt{ko}\ \ )
\end{array}
$$

$$
\begin{array}{l}
\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C};\\
\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A};\\
\qquad (\ \ \mathtt{A} \xrightarrow{\mathtt{ok}} \mathtt{C};\\
\qquad\quad \mathtt{C} \xrightarrow{\overset{\mathtt{price}}{\leftrightarrow}} \mathtt{B};\\
\qquad\quad \mathtt{C}\circ\langle\!\langle\bullet\mathtt{B};\\
\qquad\quad \mathtt{A} \xrightarrow{\mathtt{card}} \mathtt{C};\\
\qquad\quad \mathtt{B}\bullet\rangle\!\rangle\circ\mathtt{C};\\
\qquad\quad \mathtt{C} \xrightarrow{\mathtt{date}} \mathtt{A};\ \mathsf{End}\\
\qquad\quad \boxplus\\
\qquad\quad \mathtt{A} \xrightarrow{\mathtt{ko}} \mathtt{C};\ \mathsf{End}\ \ )
\end{array}
$$

Projection: Example

$$\begin{array}{l} \mathtt{A}?\,\mathtt{title}\,; \\ \mathtt{A}!\,\mathtt{price}\,; \\ \quad (\;\; \mathtt{A}?\,\mathtt{ok}\,; \\ \qquad \mathtt{B}!\overset{\mathtt{price}}{\leftrightarrow}\,; \\ \qquad \circ\langle\!\langle \bullet \mathtt{B}; \\ \\ \qquad \mathtt{B}\bullet\rangle\!\rangle\circ; \\ \qquad \mathtt{A}!\,\mathtt{date}\,; \\ \qquad + \\ \qquad \mathtt{A}?\,\mathtt{ko} \;\;) \end{array}$$

$$\begin{array}{l} \mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C}; \\ \mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A}; \\ \quad (\;\; \mathtt{A} \xrightarrow{\mathtt{ok}} \mathtt{C}; \\ \qquad \mathtt{C} \xrightarrow{\overset{\mathtt{price}}{\leftrightarrow}} \mathtt{B}; \\ \qquad \mathtt{C}\circ\langle\!\langle \bullet \mathtt{B}; \\ \qquad \mathtt{A} \xrightarrow{\mathtt{card}} \mathtt{C}; \\ \qquad \mathtt{B}\bullet\rangle\!\rangle\circ \mathtt{C}; \\ \qquad \mathtt{C} \xrightarrow{\mathtt{date}} \mathtt{A}; \mathsf{End} \\ \qquad \boxplus \\ \qquad \mathtt{A} \xrightarrow{\mathtt{ko}} \mathtt{C}; \mathsf{End} \;\;) \end{array}$$

$$\begin{array}{l} \mathtt{C}\circ\langle\!\langle \bullet; \\ \mathtt{A}?\,\mathtt{card}\,; \\ \bullet\rangle\!\rangle\circ \mathtt{C} \end{array}$$

Direct Projection

Meet

Direct Projection

Meet

$$(\Sigma_{i\in I}\mathsf{p}_i?\Lambda_i; P_i) \sqcap \mathsf{p}?\Lambda; P = \Sigma_{i\in I}\mathsf{p}_i?\Lambda_i; P_i \ + \ \mathsf{p}?\Lambda; P$$

Direct Projection

Meet

$$(\Sigma_{i\in I}\mathsf{p}_i?\Lambda_i; P_i) \sqcap \mathsf{p}?\Lambda; P = \Sigma_{i\in I}\mathsf{p}_i?\Lambda_i; P_i \ + \ \mathsf{p}?\Lambda; P$$
$$(\Sigma_{i\in I}\mathsf{p}_i?\Lambda_i; P_i) \sqcap \mathsf{p}?\Lambda; \mathsf{T} = \Sigma_{i\in I}\mathsf{p}_i?\Lambda_i; P_i$$
$$\text{if } \mathsf{p} = \mathsf{p}_j \text{ and } \Lambda = \Lambda_j \text{ and } P = P_j \text{ for some } j \in I$$

Direct Projection

Meet

$$(\Sigma_{i\in I}\mathsf{p}_i?\Lambda_i; P_i) \sqcap \mathsf{p}?\Lambda; P = \Sigma_{i\in I}\mathsf{p}_i?\Lambda_i; P_i \; + \; \mathsf{p}?\Lambda; P$$
$$(\Sigma_{i\in I}\mathsf{p}_i?\Lambda_i; P_i) \sqcap \mathsf{p}?\Lambda; \mathsf{T} = \Sigma_{i\in I}\mathsf{p}_i?\Lambda_i; P_i$$
$$\text{if } \mathsf{p} = \mathsf{p}_j \text{ and } \Lambda = \Lambda_j \text{ and } P = P_j \text{ for some } j \in I$$
$$(\Sigma_{i\in I}\mathsf{p}_i? \overset{\lambda_i}{\leftrightarrow}; P_i) \sqcap \mathbf{0} = \Sigma_{i\in I}\mathsf{p}_i? \overset{\lambda_i}{\leftrightarrow}; P_i$$

Direct Projection

Meet

$$(\Sigma_{i\in I}\mathsf{p}_i?\Lambda_i; P_i) \sqcap \mathsf{p}?\Lambda; P = \Sigma_{i\in I}\mathsf{p}_i?\Lambda_i; P_i \;+\; \mathsf{p}?\Lambda; P$$
$$(\Sigma_{i\in I}\mathsf{p}_i?\Lambda_i; P_i) \sqcap \mathsf{p}?\Lambda; \mathrm{T} = \Sigma_{i\in I}\mathsf{p}_i?\Lambda_i; P_i$$
if $\mathsf{p} = \mathsf{p}_j$ and $\Lambda = \Lambda_j$ and $P = P_j$ for some $j \in I$
$$(\Sigma_{i\in I}\mathsf{p}_i?\overset{\lambda_i}{\leftrightarrow}; P_i) \sqcap \mathbf{0} = \Sigma_{i\in I}\mathsf{p}_i?\overset{\lambda_i}{\leftrightarrow}; P_i$$
$$\mathbf{0} \sqcap \mathbf{0} = \mathbf{0}$$

Direct Projection

$$(\, \mathsf{p} \Lambda \mathsf{q}; \mathsf{G} \,) \upharpoonright \mathsf{r} \; = \begin{cases} \mathsf{q}!\Lambda; \mathsf{G} \upharpoonright \mathsf{p} & \text{if } \mathsf{r} = \mathsf{p} \\ \mathsf{p}?\Lambda; \mathsf{G} \upharpoonright \mathsf{q} & \text{if } \mathsf{r} = \mathsf{q} \\ \mathsf{G} \upharpoonright \mathsf{r} & \text{if } \mathsf{r} \notin \{\mathsf{p}, \mathsf{q}\} \end{cases}$$

Direct Projection

$$(\ \mathsf{p}\Lambda\mathsf{q}; \mathsf{G} \ ) \restriction \mathsf{r} = \begin{cases} \mathsf{q}!\Lambda; \mathsf{G} \restriction \mathsf{p} & \text{if } \mathsf{r} = \mathsf{p} \\ \mathsf{p}?\Lambda; \mathsf{G} \restriction \mathsf{q} & \text{if } \mathsf{r} = \mathsf{q} \\ \mathsf{G} \restriction \mathsf{r} & \text{if } \mathsf{r} \notin \{\mathsf{p}, \mathsf{q}\} \end{cases}$$

$$(\ \boxplus_{i \in I} \mathsf{p}\Lambda_i\mathsf{q}_i; \mathsf{G}_i \ ) \restriction \mathsf{r} = \begin{cases} \bigoplus_{i \in I}(\ \alpha_i^{\mathsf{p}}; \mathsf{G}_i \ ) \restriction \mathsf{r} & \text{if } \mathsf{r} = \mathsf{p} \\ \prod_{i \in I}(\ \alpha_i^{\mathsf{p}}; \mathsf{G}_i \ ) \restriction \mathsf{r} & \text{otherwise} \end{cases} \quad \text{where } |I| > 1$$

Direct Projection

$$(\ \mathsf{p}\Lambda\mathsf{q}; \mathsf{G} \ ) \upharpoonright \mathsf{r} \ = \begin{cases} \mathsf{q}!\Lambda; \mathsf{G} \upharpoonright \mathsf{p} & \text{if } \mathsf{r} = \mathsf{p} \\ \mathsf{p}?\Lambda; \mathsf{G} \upharpoonright \mathsf{q} & \text{if } \mathsf{r} = \mathsf{q} \\ \mathsf{G} \upharpoonright \mathsf{r} & \text{if } \mathsf{r} \notin \{\mathsf{p}, \mathsf{q}\} \end{cases}$$

$$(\ \boxplus_{i\in I} \mathsf{p}\Lambda_i \mathsf{q}_i; \mathsf{G}_i \ ) \upharpoonright \mathsf{r} \ = \begin{cases} \bigoplus_{i\in I}(\ \alpha_i^{\mathsf{p}}; \mathsf{G}_i \ ) \upharpoonright \mathsf{r} & \text{if } \mathsf{r} = \mathsf{p} \\ \prod_{i\in I}(\ \alpha_i^{\mathsf{p}}; \mathsf{G}_i \ ) \upharpoonright \mathsf{r} & \text{otherwise} \end{cases} \quad \text{where } |I| > 1$$

$$(\mu\mathbf{t}.\mathsf{G}) \upharpoonright \mathsf{p} \ = \begin{cases} \mathsf{G} \upharpoonright \mathsf{p} & \text{if } \mathbf{t} \text{ does not occur in } \mathsf{G} \\ \mu\mathbf{t}.\mathsf{G} \upharpoonright \mathsf{p} & \text{if } \mathsf{p} \in \mathsf{part}(\mathsf{G}) \\ \mathbf{0} & \text{otherwise} \end{cases}$$

Direct Projection

$$(\ \mathsf{p}\Lambda\mathsf{q}; \mathsf{G}\ ) \upharpoonright \mathsf{r} \ = \begin{cases} \mathsf{q}!\Lambda; \mathsf{G} \upharpoonright \mathsf{p} & \text{if } \mathsf{r} = \mathsf{p} \\ \mathsf{p}?\Lambda; \mathsf{G} \upharpoonright \mathsf{q} & \text{if } \mathsf{r} = \mathsf{q} \\ \mathsf{G} \upharpoonright \mathsf{r} & \text{if } \mathsf{r} \notin \{\mathsf{p}, \mathsf{q}\} \end{cases}$$

$$(\ \boxplus_{i \in I}\, \mathsf{p}\Lambda_i\mathsf{q}_i; \mathsf{G}_i\ ) \upharpoonright \mathsf{r} \ = \begin{cases} \bigoplus_{i \in I}(\ \alpha_i^{\mathsf{p}}; \mathsf{G}_i\ ) \upharpoonright \mathsf{r} & \text{if } \mathsf{r} = \mathsf{p} \\ \prod_{i \in I}(\ \alpha_i^{\mathsf{p}}; \mathsf{G}_i\ ) \upharpoonright \mathsf{r} & \text{otherwise} \end{cases} \qquad \text{where } |I| > 1$$

$$(\mu \mathbf{t}.\mathsf{G}) \upharpoonright \mathsf{p} \ = \begin{cases} \mathsf{G} \upharpoonright \mathsf{p} & \text{if } \mathbf{t} \text{ does not occur in } \mathsf{G} \\ \mu \mathbf{t}.\mathsf{G} \upharpoonright \mathsf{p} & \text{if } \mathsf{p} \in \mathsf{part}(\mathsf{G}) \\ \mathbf{0} & \text{otherwise} \end{cases} \qquad \mathbf{t} \upharpoonright \mathsf{p} \ = \mathbf{t}$$

Direct Projection

$$(\ \mathsf{p}\Lambda\mathsf{q}; \mathsf{G}\ ) \upharpoonright \mathsf{r} = \begin{cases} \mathsf{q}!\Lambda; \mathsf{G} \upharpoonright \mathsf{p} & \text{if } \mathsf{r} = \mathsf{p} \\ \mathsf{p}?\Lambda; \mathsf{G} \upharpoonright \mathsf{q} & \text{if } \mathsf{r} = \mathsf{q} \\ \mathsf{G} \upharpoonright \mathsf{r} & \text{if } \mathsf{r} \notin \{\mathsf{p}, \mathsf{q}\} \end{cases}$$

$$(\ \boxplus_{i \in I} \mathsf{p}\Lambda_i\mathsf{q}_i; \mathsf{G}_i\ ) \upharpoonright \mathsf{r} = \begin{cases} \bigoplus_{i \in I}(\ \alpha_i^{\mathsf{p}}; \mathsf{G}_i\ ) \upharpoonright \mathsf{r} & \text{if } \mathsf{r} = \mathsf{p} \\ \prod_{i \in I}(\ \alpha_i^{\mathsf{p}}; \mathsf{G}_i\ ) \upharpoonright \mathsf{r} & \text{otherwise} \end{cases} \quad \text{where } |I| > 1$$

$$(\mu\mathbf{t}.\mathsf{G}) \upharpoonright \mathsf{p} = \begin{cases} \mathsf{G} \upharpoonright \mathsf{p} & \text{if } \mathbf{t} \text{ does not occur in } \mathsf{G} \\ \mu\mathbf{t}.\mathsf{G} \upharpoonright \mathsf{p} & \text{if } \mathsf{p} \in \mathsf{part}(\mathsf{G}) \\ \mathbf{0} & \text{otherwise} \end{cases} \qquad \mathbf{t} \upharpoonright \mathsf{p} = \mathbf{t} \qquad \mathsf{End} \upharpoonright \mathsf{p} = \mathbf{0}$$

Direct Projection

$$(\ \mathsf{p}\Lambda\mathsf{q};\mathsf{G} \ ) \restriction \mathsf{r} = \begin{cases} \mathsf{q}!\Lambda;\mathsf{G}\restriction\mathsf{p} & \text{if } \mathsf{r}=\mathsf{p} \\ \mathsf{p}?\Lambda;\mathsf{G}\restriction\mathsf{q} & \text{if } \mathsf{r}=\mathsf{q} \\ \mathsf{G}\restriction\mathsf{r} & \text{if } \mathsf{r}\notin\{\mathsf{p},\mathsf{q}\} \end{cases}$$

$$(\ \boxplus_{i\in I}\,\mathsf{p}\Lambda_i\mathsf{q}_i;\mathsf{G}_i \ ) \restriction \mathsf{r} = \begin{cases} \bigoplus_{i\in I}(\ \alpha_i^{\mathsf{p}};\mathsf{G}_i \ )\restriction\mathsf{r} & \text{if } \mathsf{r}=\mathsf{p} \\ \prod_{i\in I}(\ \alpha_i^{\mathsf{p}};\mathsf{G}_i \ )\restriction\mathsf{r} & \text{otherwise} \end{cases} \quad \text{where } |I|>1$$

$$(\mu\mathbf{t}.\mathsf{G})\restriction\mathsf{p} = \begin{cases} \mathsf{G}\restriction\mathsf{p} & \text{if } \mathbf{t} \text{ does not occur in } \mathsf{G} \\ \mu\mathbf{t}.\mathsf{G}\restriction\mathsf{p} & \text{if } \mathsf{p}\in\mathsf{part}(\mathsf{G}) \\ \mathbf{0} & \text{otherwise} \end{cases} \qquad \mathbf{t}\restriction\mathsf{p} = \mathbf{t} \qquad \mathsf{End}\restriction\mathsf{p} = \mathbf{0}$$

$$(\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q};\mathsf{G})\restriction\mathsf{r} = \begin{cases} \circ\langle\!\langle\bullet\,\mathsf{q};\mathsf{G}\restriction_1(\mathsf{p},\mathsf{q}) & \text{if } \mathsf{r}=\mathsf{p} \\ \mathsf{p}\circ\langle\!\langle\bullet;\mathsf{G}\restriction_2(\mathsf{p},\mathsf{q}) & \text{if } \mathsf{r}=\mathsf{q} \\ \mathsf{G}\restriction\mathsf{r} & \text{otherwise} \end{cases}$$

Direct Projection

$$(\ \mathsf{p}\Lambda\mathsf{q}; \mathsf{G} \ ) \upharpoonright \mathsf{r} \ = \begin{cases} \mathsf{q}!\Lambda; \mathsf{G} \upharpoonright \mathsf{p} & \text{if } \mathsf{r} = \mathsf{p} \\ \mathsf{p}?\Lambda; \mathsf{G} \upharpoonright \mathsf{q} & \text{if } \mathsf{r} = \mathsf{q} \\ \mathsf{G} \upharpoonright \mathsf{r} & \text{if } \mathsf{r} \notin \{\mathsf{p}, \mathsf{q}\} \end{cases}$$

$$(\ \boxplus_{i\in I} \mathsf{p}\Lambda_i\mathsf{q}_i; \mathsf{G}_i \ ) \upharpoonright \mathsf{r} \ = \begin{cases} \bigoplus_{i\in I}(\ \alpha_i^{\mathsf{p}}; \mathsf{G}_i \ ) \upharpoonright \mathsf{r} & \text{if } \mathsf{r} = \mathsf{p} \\ \prod_{i\in I}(\ \alpha_i^{\mathsf{p}}; \mathsf{G}_i \ ) \upharpoonright \mathsf{r} & \text{otherwise} \end{cases} \quad \text{where } |I| > 1$$

$$(\mu\mathbf{t}.\mathsf{G}) \upharpoonright \mathsf{p} \ = \begin{cases} \mathsf{G} \upharpoonright \mathsf{p} & \text{if } \mathbf{t} \text{ does not occur in } \mathsf{G} \\ \mu\mathbf{t}.\mathsf{G} \upharpoonright \mathsf{p} & \text{if } \mathsf{p} \in \mathsf{part}(\mathsf{G}) \\ \mathbf{0} & \text{otherwise} \end{cases} \qquad \mathbf{t} \upharpoonright \mathsf{p} \ = \mathbf{t} \qquad \mathsf{End} \upharpoonright \mathsf{p} \ = \mathbf{0}$$

$$(\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}; \mathsf{G}) \upharpoonright \mathsf{r} \ = \begin{cases} \circ\langle\!\langle\bullet \ \mathsf{q}; \mathsf{G} \upharpoonright_1(\mathsf{p}, \mathsf{q}) & \text{if } \mathsf{r} = \mathsf{p} \\ \mathsf{p}\circ\langle\!\langle\bullet; \mathsf{G} \upharpoonright_2(\mathsf{p}, \mathsf{q}) & \text{if } \mathsf{r} = \mathsf{q} \\ \mathsf{G} \upharpoonright \mathsf{r} & \text{otherwise} \end{cases}$$

$$(\mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}; \mathsf{G}) \upharpoonright \mathsf{r} \ = \mathsf{G} \upharpoonright \mathsf{r} \qquad \text{if } \mathsf{r} \notin \{\mathsf{p}, \mathsf{q}\}$$

Delegation Projection

$$(\; r\Lambda s; G \;) \restriction_2 (p, q) = \begin{cases} s!\Lambda; G \restriction_2 (p, q) & \text{if } r = p \text{ and } s \neq q \\ r?\Lambda; G \restriction_2 (p, q) & \text{if } s = p \text{ and } r \neq q \\ G \restriction_2 (p, q) & \text{if } \{r, s\} \cap \{p, q\} = \emptyset \end{cases}$$

Delegation Projection

$$(\; r \Lambda s; G \;) \upharpoonright_2 (p, q) = \begin{cases} s!\Lambda; G \upharpoonright_2 (p, q) & \text{if } r = p \text{ and } s \neq q \\ r?\Lambda; G \upharpoonright_2 (p, q) & \text{if } s = p \text{ and } r \neq q \\ G \upharpoonright_2 (p, q) & \text{if } \{r, s\} \cap \{p, q\} = \emptyset \end{cases}$$

$$(\; r \Lambda s; G \;) \upharpoonright_1 (p, q) = G \upharpoonright_1 (p, q) \quad \text{if } r \neq q \text{ and } s \neq q$$

Delegation Projection

$$(\ r\Lambda s; G\ ) \restriction_2(p,q) = \begin{cases} s!\Lambda; G \restriction_2(p,q) & \text{if } r = p \text{ and } s \neq q \\ r?\Lambda; G \restriction_2(p,q) & \text{if } s = p \text{ and } r \neq q \\ G \restriction_2(p,q) & \text{if } \{r,s\} \cap \{p,q\} = \emptyset \end{cases}$$

$$(\ r\Lambda s; G\ ) \restriction_1(p,q) = G \restriction_1(p,q) \quad \text{if } r \neq q \text{ and } s \neq q$$

$$(q\bullet\rangle\!\rangle\circ p; G) \restriction_1(p,q) = q\bullet\rangle\!\rangle\circ; G \restriction p \qquad (q\bullet\rangle\!\rangle\circ p; G) \restriction_2(p,q) = \bullet\rangle\!\rangle\circ p; G \restriction q$$

Delegation Projection

$$(\ r\Lambda s; G \ ) \restriction_2(p,q) \ = \begin{cases} s!\Lambda; G \restriction_2(p,q) & \text{if } r = p \text{ and } s \neq q \\ r?\Lambda; G \restriction_2(p,q) & \text{if } s = p \text{ and } r \neq q \\ G \restriction_2(p,q) & \text{if } \{r,s\} \cap \{p,q\} = \emptyset \end{cases}$$

$$(\ r\Lambda s; G \ ) \restriction_1(p,q) \ = G \restriction_1(p,q) \quad \text{if } r \neq q \text{ and } s \neq q$$

$$(q\bullet\rangle\!\rangle\circ p; G) \restriction_1(p,q) \ = q\bullet\rangle\!\rangle\circ; G \restriction p \qquad (q\bullet\rangle\!\rangle\circ p; G) \restriction_2(p,q) \ = \ \bullet\rangle\!\rangle\circ p; G \restriction q$$

$$(r\circ\langle\!\langle\bullet s; G) \restriction_1(p,q) \ = (r\bullet\rangle\!\rangle\circ s; G) \restriction_1(p,q) \ = G \restriction_1(p,q) \quad \text{if } \ \{r,s\} \cap \{p,q\} = \emptyset$$

Delegation Projection

$$(\; r\Lambda s; G \;) \restriction_2 (p, q) \; = \begin{cases} s!\Lambda; G \restriction_2 (p, q) & \text{if } r = p \text{ and } s \neq q \\ r?\Lambda; G \restriction_2 (p, q) & \text{if } s = p \text{ and } r \neq q \\ G \restriction_2 (p, q) & \text{if } \{r, s\} \cap \{p, q\} = \emptyset \end{cases}$$

$$(\; r\Lambda s; G \;) \restriction_1 (p, q) \; = G \restriction_1 (p, q) \;\; \text{if } r \neq q \text{ and } s \neq q$$

$$(q\bullet\rangle\!\rangle\circ p; G) \restriction_1 (p, q) \; = q\bullet\rangle\!\rangle\circ; G \restriction p \qquad (q\bullet\rangle\!\rangle\circ p; G) \restriction_2 (p, q) \; = \; \bullet\rangle\!\rangle\circ p; G \restriction q$$

$$(r\circ\langle\!\langle\bullet s; G) \restriction_1 (p, q) \; = (r\bullet\rangle\!\rangle\circ s; G) \restriction_1 (p, q) \; = G \restriction_1 (p, q) \;\; \text{if } \; \{r, s\} \cap \{p, q\} = \emptyset$$

$$(r\circ\langle\!\langle\bullet s; G) \restriction_2 (p, q) \; = (r\bullet\rangle\!\rangle\circ s; G) \restriction_2 (p, q) \; = G \restriction_2 (p, q) \;\; \text{if } \; \{r, s\} \cap \{p, q\} = \emptyset$$

Typing Rule

$$\dfrac{\begin{array}{ll} \mathsf{q}_i \bullet\rangle\!\rangle \circ; P_i \leq \mathsf{G}\restriction_1(\mathsf{p}_i, \mathsf{q}_i) & (i \in I) \\ Q_i \leq \mathsf{G}\restriction_2(\mathsf{p}_i, \mathsf{q}_i) & (i \in I) \\ R_j \leq \mathsf{G}\restriction \mathsf{r}_j & (j \in J) \\ \multicolumn{2}{l}{\mathsf{part}(\mathsf{G}) \subseteq \{\mathsf{p}_i \mid i \in I\} \cup \{\mathsf{q}_i \mid i \in I\} \cup \{\mathsf{r}_j \mid j \in J\} \text{ all participants distinct}} \end{array}}{\vdash \Pi_{i \in I} \overset{*}{\mathsf{p}}_i[\![\, \mathsf{q}_i \bullet\rangle\!\rangle \circ; P_i \,]\!] \parallel \Pi_{i \in I}\, \mathsf{p}_i[\![\, Q_i \,]\!] \parallel \Pi_{j \in J}\, \mathsf{r}_j[\![\, R_j \,]\!] : \mathsf{G}}$$

Typing Rule

$$\dfrac{\begin{array}{ll} \mathsf{q}_i \bullet\!\rangle\!\rangle \circ; P_i \leq \mathsf{G}\upharpoonright_1(\mathsf{p}_i, \mathsf{q}_i) & (i \in I) \\ Q_i \leq \mathsf{G}\upharpoonright_2(\mathsf{p}_i, \mathsf{q}_i) & (i \in I) \\ R_j \leq \mathsf{G}\upharpoonright \mathsf{r}_j & (j \in J) \\ \mathrm{part}(\mathsf{G}) \subseteq \{\mathsf{p}_i \mid i \in I\} \cup \{\mathsf{q}_i \mid i \in I\} \cup \{\mathsf{r}_j \mid j \in J\} & \text{all participants distinct} \end{array}}{\vdash \Pi_{i \in I} \overset{*}{\mathsf{p}_i}[\![\, \mathsf{q}_i \bullet\!\rangle\!\rangle \circ; P_i \,]\!] \parallel \Pi_{i \in I}\, \mathsf{p}_i[\![\, Q_i \,]\!] \parallel \Pi_{j \in J}\, \mathsf{r}_j[\![\, R_j \,]\!] : \mathsf{G}}$$

$$\mathtt{A}[\![\, \mathtt{C}!\,\mathtt{card}\,;\mathtt{C}?\,\mathtt{date} \,]\!] \parallel \overset{*}{\mathtt{C}}[\![\, \mathtt{B} \bullet\!\rangle\!\rangle \circ; \mathtt{A}!\,\mathtt{date} \,]\!] \parallel \mathtt{C}[\![\, \mathtt{A}?\,\mathtt{card}\,; \bullet\!\rangle\!\rangle \circ \mathtt{C} \,]\!]$$

Typing Rule

$$\dfrac{\begin{array}{ll} \mathsf{q}_i\bullet\!\rangle\!\rangle\!\circ; P_i \leq \mathsf{G}\!\restriction_1\!(\mathsf{p}_i,\mathsf{q}_i) & (i \in I) \\ Q_i \leq \mathsf{G}\!\restriction_2\!(\mathsf{p}_i,\mathsf{q}_i) & (i \in I) \\ R_j \leq \mathsf{G}\!\restriction\!\mathsf{r}_j & (j \in J) \end{array} \\ \mathsf{part}(\mathsf{G}) \subseteq \{\mathsf{p}_i \mid i \in I\} \cup \{\mathsf{q}_i \mid i \in I\} \cup \{\mathsf{r}_j \mid j \in J\} \text{ all participants distinct}}{\vdash \Pi_{i\in I}\overset{*}{\mathsf{p}_i}[\![\, \mathsf{q}_i\bullet\!\rangle\!\rangle\!\circ; P_i \,]\!] \parallel \Pi_{i\in I}\, \mathsf{p}_i[\![\, Q_i \,]\!] \parallel \Pi_{j\in J}\, \mathsf{r}_j[\![\, R_j \,]\!] : \mathsf{G}}$$

$$\mathtt{A}[\![\, \mathtt{C!\,card\,;C?\,date} \,]\!] \parallel \overset{*}{\mathtt{C}}[\![\, \mathtt{B}\bullet\!\rangle\!\rangle\!\circ; \mathtt{A!\,date} \,]\!] \parallel \mathtt{C}[\![\, \mathtt{A?\,card\,;} \bullet\!\rangle\!\rangle\!\circ\mathtt{C} \,]\!]$$

$$\begin{array}{l} \mathtt{A} \xrightarrow{\mathtt{card}} \mathtt{C}; \\ \mathtt{B}\bullet\!\rangle\!\rangle\!\circ\mathtt{C}; \\ \mathtt{C} \xrightarrow{\mathtt{date}} \mathtt{A}; \mathsf{End} \end{array}$$

Subject Reduction

If $\vdash \mathbb{N} : \mathsf{G}$ and $\mathbb{N} \xrightarrow{\phi} \mathbb{N}'$, then $\vdash \mathbb{N}' : \mathsf{G}'$ for some G'.

Session Fidelity

- If $\vdash \mathbb{N} : \mathsf{G}$ and $\mathbb{N} \xrightarrow{\mathsf{p}\Lambda\mathsf{q}} \mathbb{N}'$, then $\mathsf{G} = \phi_1; \ldots; \phi_n; (\boxplus_{i \in I} \mathsf{p}\Lambda_i\mathsf{q}_i; \mathsf{G}_i \boxplus \mathsf{p}\Lambda\mathsf{q}; \mathsf{G}')$, where ϕ_j for $1 \leq j \leq n$ is an atomic interaction not involving p and q.

Session Fidelity

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- If $\vdash \mathbb{N} : \mathsf{G}$ and $\mathbb{N} \xrightarrow{\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}} \mathbb{N}'$, then $\mathsf{G} = \phi_1; \ldots; \phi_n; \mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}; \mathsf{G}'$, where ϕ_i for $1 \leq i \leq n$ is an atomic interaction not involving p and q.

Session Fidelity

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- If $\vdash \mathbb{N} : \mathsf{G}$ and $\mathbb{N} \xrightarrow{\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}} \mathbb{N}'$, then $\mathsf{G} = \phi_1; \ldots; \phi_n; \mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}; \mathsf{G}'$, where ϕ_i for $1 \leq i \leq n$ is an atomic interaction not involving p and q.
- If $\vdash \mathbb{N} : \mathsf{G}$ and $\mathbb{N} \xrightarrow{\mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}} \mathbb{N}'$, then $\mathsf{G} = \phi_1; \ldots; \phi_n; \mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}; \mathsf{G}'$, where ϕ_i for $1 \leq i \leq n$ is an atomic interaction not involving p and q.

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- If $\vdash \mathbb{N} : \mathsf{G}$ and $\mathbb{N} \xrightarrow{\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}} \mathbb{N}'$, then $\mathsf{G} = \phi_1; \ldots; \phi_n; \mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}; \mathsf{G}'$, where ϕ_i for $1 \leq i \leq n$ is an atomic interaction not involving p and q.
- If $\vdash \mathbb{N} : \mathsf{G}$ and $\mathbb{N} \xrightarrow{\mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}} \mathbb{N}'$, then $\mathsf{G} = \phi_1; \ldots; \phi_n; \mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}; \mathsf{G}'$, where ϕ_i for $1 \leq i \leq n$ is an atomic interaction not involving p and q.
- If $\vdash \mathbb{N} : \boxplus_{i\in I}\mathsf{p}\Lambda_i\mathsf{q}_i; \mathsf{G}_i$, then $\mathbb{N} = \mathsf{p}[\![\, \oplus_{i\in I'}\mathsf{q}_i!\Lambda_i; P_i \,]\!] \parallel \mathbb{N}_0$ with $I' \subseteq I$ and $\mathbb{N} \xrightarrow{\mathsf{p}\Lambda_i\mathsf{q}_i} \mathbb{N}_i$ and $\vdash \mathbb{N}_i : \mathsf{G}_i$ for all $i \in I'$.

Session Fidelity

- If $\vdash \mathbb{N} : \mathsf{G}$ and $\mathbb{N} \xrightarrow{\mathsf{p}\Lambda\mathsf{q}} \mathbb{N}'$, then $\mathsf{G} = \phi_1; \ldots; \phi_n; (\boxplus_{i \in I} \mathsf{p}\Lambda_i\mathsf{q}_i; \mathsf{G}_i \boxplus \mathsf{p}\Lambda\mathsf{q}; \mathsf{G}')$, where ϕ_j for $1 \leq j \leq n$ is an atomic interaction not involving p and q.
- If $\vdash \mathbb{N} : \mathsf{G}$ and $\mathbb{N} \xrightarrow{\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}} \mathbb{N}'$, then $\mathsf{G} = \phi_1; \ldots; \phi_n; \mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}; \mathsf{G}'$, where ϕ_i for $1 \leq i \leq n$ is an atomic interaction not involving p and q.
- If $\vdash \mathbb{N} : \mathsf{G}$ and $\mathbb{N} \xrightarrow{\mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}} \mathbb{N}'$, then $\mathsf{G} = \phi_1; \ldots; \phi_n; \mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}; \mathsf{G}'$, where ϕ_i for $1 \leq i \leq n$ is an atomic interaction not involving p and q.
- If $\vdash \mathbb{N} : \boxplus_{i \in I}\mathsf{p}\Lambda_i\mathsf{q}_i; \mathsf{G}_i$, then $\mathbb{N} = \mathsf{p}[\![\oplus_{i \in I'} \mathsf{q}_i!\Lambda_i; P_i]\!] \parallel \mathbb{N}_0$ with $I' \subseteq I$ and $\mathbb{N} \xrightarrow{\mathsf{p}\Lambda_i\mathsf{q}_i} \mathbb{N}_i$ and $\vdash \mathbb{N}_i : \mathsf{G}_i$ for all $i \in I'$.
- If $\vdash \mathbb{N} : \phi; \mathsf{G}$, then $\mathbb{N} \xrightarrow{\phi} \mathbb{N}'$ and $\vdash \mathbb{N}' : \mathsf{G}$.

Strong Progress

- If $\mathbb{N} = \mathsf{p}[\![\, \oplus_{i \in I} \mathsf{q}_i!\Lambda_i; P_i \,]\!] \parallel \mathbb{N}_0$, then $\mathbb{N} \xrightarrow{\vec{\phi}\ \mathsf{p}\Lambda_i\mathsf{q}_i} \mathbb{N}'$ for some $\vec{\phi}$ and for all $i \in I$.

Strong Progress

- If $\mathbb{N} = \mathsf{p}[\![\, \oplus_{i \in I} \mathsf{q}_i!\Lambda_i; P_i \,]\!] \parallel \mathbb{N}_0$, then $\mathbb{N} \xrightarrow{\vec{\phi}\ \mathsf{p}\Lambda_i\mathsf{q}_i} \mathbb{N}'$ for some $\vec{\phi}$ and for all $i \in I$.
- If $\mathbb{N} = \mathsf{p}[\![\, \Sigma_{i \in I} \mathsf{q}_i?\lambda_i; P_i \,]\!] \parallel \mathbb{N}_0$, then $\mathbb{N} \xrightarrow{\vec{\phi}\ \mathsf{q}_i\lambda_i\mathsf{p}} \mathbb{N}'$ for some $\vec{\phi}$ and for some $i \in I$.

Strong Progress

- If $\mathbb{N} = \mathsf{p}[\![\, \oplus_{i\in I}\mathsf{q}_i!\Lambda_i; P_i \,]\!] \parallel \mathbb{N}_0$, then $\mathbb{N} \xrightarrow{\overrightarrow{\phi}\ \mathsf{p}\Lambda_i\mathsf{q}_i} \mathbb{N}'$ for some $\overrightarrow{\phi}$ and for all $i \in I$.
- If $\mathbb{N} = \mathsf{p}[\![\, \Sigma_{i\in I}\mathsf{q}_i?\lambda_i; P_i \,]\!] \parallel \mathbb{N}_0$, then $\mathbb{N} \xrightarrow{\overrightarrow{\phi}\ \mathsf{q}_i\lambda_i\mathsf{p}} \mathbb{N}'$ for some $\overrightarrow{\phi}$ and for some $i \in I$.
- If $\mathbb{N} = \mathsf{p}[\![\, \circ\langle\!\langle \bullet\mathsf{q}; P \,]\!] \parallel \mathbb{N}_0$, then $\mathbb{N} \xrightarrow{\overrightarrow{\phi}\ \mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}\ \overrightarrow{\phi'}\ \mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}} \mathbb{N}'$ for some $\overrightarrow{\phi}$ and $\overrightarrow{\phi'}$.

Strong Progress

- If $\mathbb{N} = \mathsf{p}[\![\, \oplus_{i\in I}\mathsf{q}_i!\Lambda_i; P_i \,]\!] \parallel \mathbb{N}_0$, then $\mathbb{N} \xrightarrow{\overrightarrow{\phi}\ \mathsf{p}\Lambda_i\mathsf{q}_i} \mathbb{N}'$ for some $\overrightarrow{\phi}$ and for all $i \in I$.
- If $\mathbb{N} = \mathsf{p}[\![\, \Sigma_{i\in I}\mathsf{q}_i?\lambda_i; P_i \,]\!] \parallel \mathbb{N}_0$, then $\mathbb{N} \xrightarrow{\overrightarrow{\phi}\ \mathsf{q}_i\lambda_i\mathsf{p}} \mathbb{N}'$ for some $\overrightarrow{\phi}$ and for some $i \in I$.
- If $\mathbb{N} = \mathsf{p}[\![\, \circ\langle\!\langle \bullet\mathsf{q}; P \,]\!] \parallel \mathbb{N}_0$, then $\mathbb{N} \xrightarrow{\overrightarrow{\phi}\ \mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}\ \overrightarrow{\phi'}\ \mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}} \mathbb{N}'$ for some $\overrightarrow{\phi}$ and $\overrightarrow{\phi'}$.
- If $\mathbb{N} = \mathsf{q}[\![\, \mathsf{p}\circ\langle\!\langle \bullet; Q \,]\!] \parallel \mathbb{N}_0$, then $\mathbb{N} \xrightarrow{\overrightarrow{\phi}\ \mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}\ \overrightarrow{\phi'}\ \mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}} \mathbb{N}'$ for some $\overrightarrow{\phi}$ and $\overrightarrow{\phi'}$.

Internal versus Channel Delegation

- pro

Internal versus Channel Delegation

- pro
  - internal delegation allows a better control of the whole conversation

Internal versus Channel Delegation

- pro
  - internal delegation allows a better control of the whole conversation
  - internal delegation assures progress with a simple type system

Internal versus Channel Delegation

- pro
  - internal delegation allows a better control of the whole conversation
  - internal delegation assures progress with a simple type system
- con

Internal versus Channel Delegation

- pro
  - internal delegation allows a better control of the whole conversation
  - internal delegation assures progress with a simple type system
- con
  - channel delegation can represent more protocols

Future Work

- global types allowing

Future Work

- global types allowing
  - nested delegation

Future Work

- global types allowing
  - nested delegation
  - deputies to make choices

Future Work

- global types allowing
  - nested delegation
  - deputies to make choices
  - ...

Future Work

- global types allowing
  - nested delegation
  - deputies to make choices
  - ...
- coherence of sets of session types

Future Work

- global types allowing
  - nested delegation
  - deputies to make choices
  - ...
- coherence of sets of session types
- integration with reversibility

Related Papers

- Kohei Honda, Nobuko Yoshida, and Marco Carbone. Multiparty asynchronous session types. *Journal of the ACM*, 63(1):9, 2016.

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- Raymond Hu and Nobuko Yoshida. Explicit connection actions in multiparty session types. In *FASE*, volume 10202 of *LNCS*, pages 116–133. Springer, 2017.
- Alceste Scalas, Ornela Dardha, Raymond Hu, and Nobuko Yoshida. A linear decomposition of multiparty sessions for safe distributed programming. In *ECOOP*, volume 74 of *LIPIcs*, pages 24:1–24:31. Schloss Dagstuhl, 2017.

Questions

Thank you


obrigado
Dank U
Merci
mahalo
Köszi
спасибо
Grazie
Thank you
mauruuru
Takk
Gracias
Dziękuję
Děkuju
danke
Kiitos