Which Energy Minimization for my MRF/CRF? A Cheat-Sheet

Created in the Reading Club on “Energy Minimization in Computer Vision”
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	situation			results/output				
Algorithm[a]	states	topology	pairwise	marginals	convergence	optimal	complexity	references
Plain BP	(m)any	tree	any	yes	yes	yes	$O(nh^2)$	[4, 1]
Loopy BP	(m)any	any	any	yes	no	no	$O(eh^2i)$	[4, 1]
truncation trick for BP	(m)any	tree/any	truncated	yes	yes/no	yes/no	$O(nhk)/O(ehik))$	[7]
distance transform for BP	ordered[b][j]	tree/any	limited[c]	no	yes/no	yes/no	$O(nh)/O(ehi)$	[5]
TRW-S	any	any	any	yes[d]	yes	no[e]	$O(eh^2i)$ [f]	[6]
on trees	any	tree	any	⇒ same as plain BP				
on 2-state grids	2	grid	submodular	⇒ same as Graphcut				
Graphcut	2[j]	any	submodular	no	yes	yes		
2-state augmenting path Dinic	2[j]	any	submodular	no	yes	yes	$O(n^2e)$	
2-state push-relabel FordFulkerson	2[j]	any	submodular	no	yes	yes	$O(e^2U)$	
2-state Boykov	2[j]	any[g]	submodular	no	yes	yes	$O(n^2e\|C\|)$[h]	[2]
α/β swap	few[j]	any	semi-metric	no	yes	no	$O(h^2B^{\alpha\beta}i)$	[3]
α-expansion	few[j]	any	metric	no	yes	no[i]	$O(hBi)$	[3]

[a] This is not an exhaustive list. There are other methods such as MCMC, simulated annealing, and linear programming that can also be used in some cases.
[b] grid-like in $\geq$ 1D
[c] linear-combinations and/or min-combinations of L_1, L_2, box
[d] a way to get something like a marginal is described in [8]
[e] TRW-S also outputs a lower bound on the energy which can be used to determine if the found solution is optimal
[f] there also is an averaging step in this algorithm, which takes some (non-significant) time
[g] most advantageous with low-connected grid-graphs
[h] in practice for vision problems often very fast
[i] but the energy of the solution is within a known factor of the global optimum
[j] SSFN = same states for all nodes. Denotes that the individual nodes must share the same state space.

Description of the columns

states	conditions on the state space of the nodes
topology	conditions on the topology of the graph, i.e. which pairs of nodes have a pairwise term
pairwise	conditions on the form of the pairwise term
marginals	does the algorithm produce marginals?
convergence	is the algorithm guaranteed to converge?
optimal	is the configuration the algorithm determines guaranteed to be an optimal one?
complexity	the computational worst-case complexity of the algorithm

Notation

(m)any	works for any number of states, but it is especially useful for many, typically >100
few	works in principle for any number of states, but especially useful with 2 to 32 (segmentation/stereo); never used with > 256 (denoising)
n	number of nodes
h	number of states of the nodes
e	number of edges. For 4-connected grids $e = n$ (typical in many computer vision applications).
i	number of iterations
k	is a constant, typically much smaller than h, equal to the number of states covered by the truncated pairwise term (i.e. the area of the pairwise term)
U	maximum edge weight
$\|C\|$	cost of the minimal cut
$B^{\alpha\beta}$	the cost of the graph cut algorithm which is used as a subroutine on the graph containing only of the nodes with states α and β
B	the cost of the graph cut algorithm which is used as a subroutine (on a graph with as many nodes as the original graph)

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