Two Sides of the Same Coin: Session Types and Game Semantics

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ABCD meeting
17th December 2018

Protocols: two formalisations

A simple FTP protocol:

1. Connect to Server and issue a request for a file
2. Server answers either `notfound`, or `found` and a port where to download the file.
3. At any time Client can close the connection.

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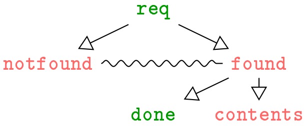
Prot.	$!\mathtt{req}.\ \& \begin{cases} ?\mathtt{notfound}; \\ ?\mathtt{found}(?\mathtt{content} : \mathtt{String}). \\ \quad !\mathtt{done} \end{cases}$	

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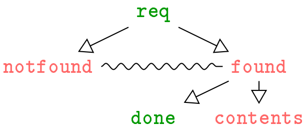
Prot.	$!\mathtt{req}.\ \& \begin{cases} ?\mathtt{notfound}; \\ ?\mathtt{found}(?\mathtt{content} : \mathtt{String}). \\ \quad !\mathtt{done} \end{cases}$	
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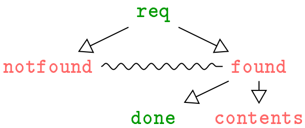
Prot.	$!\mathtt{req}.\ \& \begin{cases} ?\mathtt{notfound}; \\ ?\mathtt{found}(?\mathtt{content} : \mathtt{String}). \\ \quad !\mathtt{done} \end{cases}$	
Impl.	Processes	Strategies

Striking similarity but two different uses:

- **Session Types**: analysis of message-passing concurrency.
- **Game Semantics**: study of higher-order programs.

Our contributions

Despite the similarity, a discrepancy:

- processes: **synchronous** strategies: **asynchronous**

$\rightsquigarrow$ Solved by the introduction of **coincident strategies**: a model of synchronous and concurrent computation.

Our **key contribution**: a correspondence

session types $\leftrightarrow$ games recursive, forest-like

processes $\rightarrow$ coincident strategies

asyncronous processes $\leftrightarrow$ asynchronous strategies confusion-free

The session π-calculus [Hon93]

$$
\begin{array}{rcll}
S, T & ::= & \mathsf{end} & \textbf{Channel types} \\
& | & \bigoplus_{i \in I} \{!\ell_i \langle \vec{S_i} \rangle . T_i\} & \textit{Output/Selection} \\
& | & \&_{i \in I} \{?\ell_i (\vec{S_i}) . T_i\} & \textit{Input/Branching} \\
P, Q & ::= & \mathbf{0} & \textbf{Processes} \\
& | & (P \parallel Q) \mid (\nu a) P & \\
& | & a!\ell \langle u_1, \ldots, u_n \rangle . P & \textit{Selection/Output on } a \\
& | & a \,\&\, \{?\ell_i (\vec{x_i}) . P_i\} & \textit{Branching/Input on } a
\end{array}
$$

(The paper has recursive types and processes, as well as nondeterministic choices.)

Typing judgements. $P \triangleright a_1 : T_1, \ldots, a_n : T_n$.

$\leadsto$ Linear discipline ensures the absence of races.

Contextual equivalence. Barbed congruence: $P \approx Q$.

Game semantics: an introduction [HO00, AJM00]

Operational semantics of **higher-order open programs**.

Program

$f : \mathbb{N} \to \mathbb{N} \vdash f\, \mathsf{tt} : \mathbb{N}$

border

Context

$[][f := \lambda b.\, \mathbf{not}\ b]$

$[\![f 3]\!]$

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Operational semantics of **higher-order open programs**.

Program $f : \mathbb{N} \to \mathbb{N} \vdash f\,\mathrm{tt} : \mathbb{N}$ $\xleftarrow{\mathtt{go}(\alpha, \kappa_1)}$ Context $[][f := \lambda b.\, \textbf{not}\ b]$

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$\mathtt{go}(\alpha, \kappa_1)$

$[\![f\,3]\!] \ni$

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$$\underset{f : \mathbb{N} \to \mathbb{N} \vdash f\,\mathsf{tt} : \mathbb{N}}{\text{Program}} \xrightarrow{\mathtt{call}(\alpha, (\mathsf{tt}, \kappa_2))} \underset{[][f := \lambda b.\, \mathbf{not}\ b]}{\text{Context}}$$

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$$[\![f\,3]\!] \ni \quad \begin{array}{c} \mathtt{go}(\alpha, \kappa_1) \\ \downarrow \\ \mathtt{call}(\alpha, (\mathsf{tt}, \kappa_2)) \end{array}$$

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Program $f : \mathbb{N} \to \mathbb{N} \vdash f\,\mathsf{tt} : \mathbb{N}$ $\xleftarrow{\texttt{ret}(\kappa_2, \mathsf{ff})}$ Context $[\,][f := \lambda b.\,\mathbf{not}\ b]$

border

$[\![f\,3]\!] \ni$ $\texttt{go}(\alpha, \kappa_1) \to \texttt{call}(\alpha, (\mathsf{tt}, \kappa_2)) \to \texttt{ret}(\kappa_2, \mathsf{ff})$

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Program $\qquad\qquad\qquad \texttt{ret}(\kappa_1, \mathsf{ff}) \qquad\qquad\qquad$ Context

$$f : \mathbb{N} \to \mathbb{N} \vdash f\,\mathsf{tt} : \mathbb{N} \xrightarrow{\texttt{ret}(\kappa_1, \mathsf{ff})} [\,][f := \lambda b.\,\mathbf{not}\ b]$$

border

$$[\![f\,3]\!] \ni \quad \begin{array}{c} \texttt{go}(\alpha, \kappa_1) \\ \downarrow \\ \texttt{call}(\alpha, (\mathsf{tt}, \kappa_2)) \\ \downarrow \\ \texttt{ret}(\kappa_2, \mathsf{ff}) \\ \downarrow \\ \texttt{ret}(\kappa_1, \mathsf{ff}) \end{array}$$

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Context

$[\,][f := \lambda b.\,\mathbf{not}\ b]$

$$[\![f\,3]\!] \ni \begin{array}{c} \mathtt{go} \\ \downarrow \\ \mathtt{call(tt)} \\ \downarrow \\ \mathtt{ret(ff)} \\ \downarrow \\ \mathtt{ret(ff)} \end{array}$$

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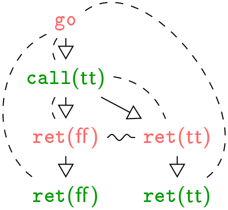

Game semantics: an introduction [HO00, AJM00]

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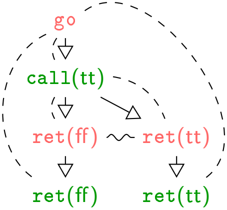
$[][f := \lambda b.\,\mathbf{not}\ b]$

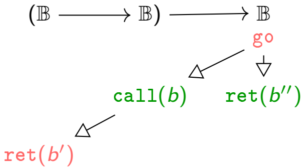
border

The **protocol** (depending on the border type) is a **game**.

$[\![f\,3]\!] =$



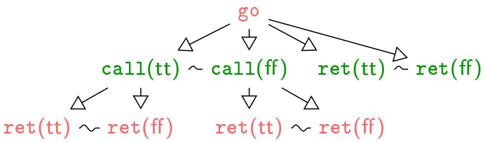
on



Game semantics on event structures [RW11]

Games: polarised event structures $(A, \leq_A, \sim, \lambda \colon A \to \{-, +\})$

- Causality ($\leq_A$) and conflict ($\sim$) represents **rules**.



A

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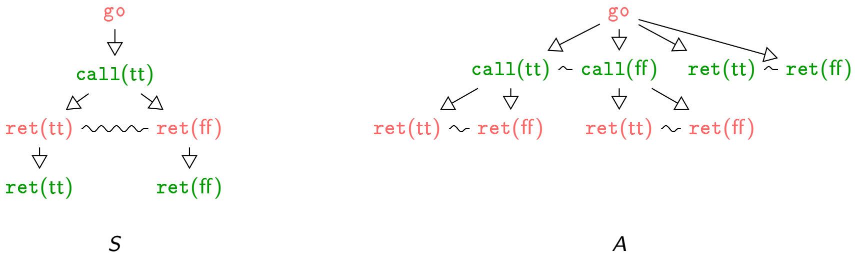
Strategies on A: event structures S



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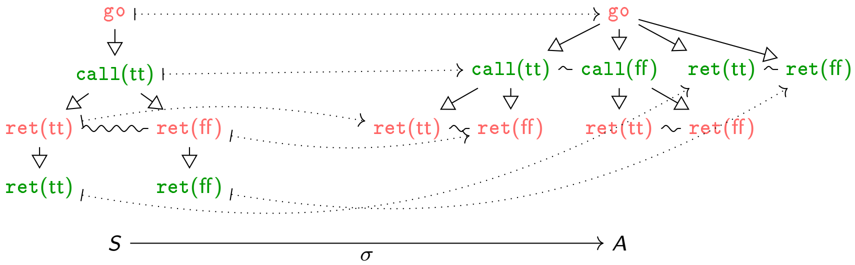
Strategies on A: event structures S with $\sigma : S \to A$.

- Conditions to ensure the respect of rules of A.



Interpretation of types as games

$$[\![\mathop{\&}_{i\in I} ?\ell_i(\vec{S_i}).T_i]\!] = \begin{tikzcd}[column sep=small] & \ell_1 \arrow[dl] \arrow[d] \arrow[dr] \arrow[rrr, squiggly, dash] & & & \ell_2 \arrow[dl] \arrow[d] \arrow[dr] \arrow[r, squiggly, dash] & \dots \\ [\![S_1^1]\!] & \dots & [\![T_1]\!] & [\![S_2^1]\!] & \dots & [\![T_2]\!] \end{tikzcd}$$

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$$[\![s_1 : S_1, \dots, s_n : S_n]\!] = [\![S_1]\!] \parallel \dots \parallel [\![S_n]\!]$$

- **Image** of the interpretation: (recursive) forest-like games.
  $\rightsquigarrow$ Games used in semantics arise from session types.
- Interpretation gets rid of irrelevant syntactic information:

$$[\![?\ell(S,T).U]\!] \cong [\![?\ell(U,S).T]\!].$$

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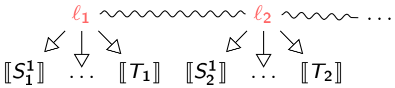

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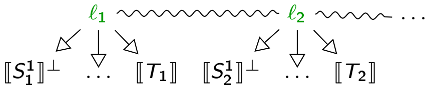

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$\rightsquigarrow$ Next step: Interpret $P \triangleright \Gamma$ as a strategy on $[\![\Gamma]\!]$.

The main issue: name-passing

How to interpret $a!\ell\langle b\rangle \triangleright a : !\ell\langle S\rangle, b : S$?

$$\begin{array}{ccc} a : !\ell & \qquad & [\![S]\!] \\ \downarrow & & \\ [\![S]\!]^{\perp} & & \end{array}$$

$$[\![a : !\ell\langle S\rangle, b : S]\!]$$

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$$\begin{tikzcd}
a : !\ell \arrow[d] & \mathtt{x} \arrow[d] \\
\mathtt{x} \arrow[d] & \mathtt{y} \\
\mathtt{y} &
\end{tikzcd}$$

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Introduces an **asynchronous** forwarder, inducing a **delay**
$\rightsquigarrow$ Delay observable using **synchrony**.

$$[\![a!\ell\langle b_1, b_2\rangle.\ \ b_1!\ell_1.\ \ b_2!\ell_2\ ]\!]_{\mathsf{async}} \approx [\![a!\ell\langle b_1, b_2\rangle.(b_1!\ell_1 \parallel b_2!\ell_2)]\!]_{\mathsf{async}}.$$

The model is not **adequate**!

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To avoid this delay, we need **simultaneity**.
$\rightsquigarrow$ Simultaneity can be encoded using **causal loops** [GM11].

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Coincident strategies and causal loops

Definition

A **coincident event structure** is a triple $(E, \leq, \sim)$ where $\leq$ is a preorder, satisfying the usual axioms of event structures.

A **coincidence** is an equivalence class for $\simeq := (\leq \cap \geq)$.

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A **coincidence** is an equivalence class for $\simeq := (\leq \cap \geq)$.

Definition

A **coincident strategy** on a game A is a coincident event structure S and a map $\sigma : S \to A$ such that coincidence of S are of the form $\{s\}$ or $\{s, s'\}$.

Theorem

There is a compact-closed category of coincident strategies.

$\rightsquigarrow$ Composition needs to tell apart deadlocks and coincidences.

Interpretation of processes as coincident strategies

By induction using the **synchronous forwarder**: $\mathbb{C}\mathbb{C}_A$.

$$\begin{tikzcd} \mathtt{x} \arrow[r, dash] \arrow[d] & \mathtt{x} \arrow[d] \\ \mathtt{y} \arrow[r, dash] & \mathtt{y} \end{tikzcd} \qquad\qquad \begin{tikzcd} \mathtt{x} \arrow[d] & \mathtt{x} \arrow[d] \\ \mathtt{y} & \mathtt{y} \end{tikzcd}$$

$$\mathbb{C}\mathbb{C}_A \qquad\qquad\qquad A^\perp \parallel A$$

A few cases:

$$[\![(\nu a : A)P]\!] = \mathbb{C}\mathbb{C}_A \odot [\![P]\!]$$

$$[\![a!\ell\langle\vec{b}\rangle.P]\!] = \ell \cdot ([\![P]\!] \parallel \mathbb{C}\mathbb{C}_{[\![\vec{B}]\!]}) \qquad \text{where } \vec{b} : \vec{B}$$

$\leadsto$ Every typed process $P \triangleright \Gamma$ becomes a strategy $[\![P]\!]$ on $[\![\Delta]\!]$.

Properties of the model

$$\text{Processes} \xrightarrow{\textbf{Fully abstract interpretation}} \textbf{Coincident strat.}$$

- Interpretation of processes is (intensionally) fully abstract:

$$P \approx Q \qquad \Leftrightarrow \qquad [\![P]\!] \approx [\![Q]\!].$$

- In both settings, barbed congruence is characterised by **weak bisimulation**.

Properties of the model

$$\begin{tikzcd}[column sep=huge]
\text{Processes} \arrow[r, "\textbf{Fully abstract interpretation}"] & \textbf{Coincident strat.} \arrow[d, bend left, "\substack{\textbf{Encoding with}\\ \textbf{acknowledgements}}"] \\
& \text{Asynchronous strat. [MM07, RW11]}
\end{tikzcd}$$

- Define an translation on games:

$$\begin{array}{ll}
A & \mapsto \uparrow A \\
\quad \mathtt{x} \mathrel{-\!\triangleright} \mathtt{y} & \mapsto \quad \mathtt{x} \mathrel{-\!\triangleright} \mathtt{ack}_{\mathtt{x}} \mathrel{-\!\triangleright} \mathtt{y} \mathrel{-\!\triangleright} \mathtt{ack}_{\mathtt{y}}
\end{array}$$

- Lift this translation on strategies: $S : A \mapsto \uparrow S : \uparrow A$.
- Translation is **injective** (but not fully abstract).

Properties of the model

$$
\begin{tikzcd}
\text{Processes} \arrow[r, "\text{Fully abstract interpretation}"] & \text{Coincident strat.} \arrow[d, bend left, "\text{Encoding with acknowledgements}"] \\
\text{Courteous processes} & \text{Asynchronous strat. [MM07, RW11]} \arrow[l, bend left, "\text{Finite definability}"]
\end{tikzcd}
$$

Show how to define coincidence-free strategies as processes:

$$
\begin{tikzcd}
 & \ell_3 & \\
\ell_1 \arrow[ur] & & \ell_2 \arrow[ul]
\end{tikzcd}
\quad \rightsquigarrow (\nu d)(a?\ell_1.\, d! \mid d?.\, b?\ell_2.\, c!\ell_3)
$$

Properties of the model

$$\begin{tikzcd}
\text{Processes} \arrow[rr, "\textbf{Fully abstract interpretation}"] \arrow[d, dotted, bend right] & & \textbf{Coincident strat.} \arrow[d, bend left, "\textbf{Encoding with acknowledgements}"] \\
\text{Courteous processes} & & \text{Asynchronous strat. [MM07, RW11]} \arrow[ll, bend left, "\textbf{Finite definability}"]
\end{tikzcd}$$

By composition, we get a translation on processes that replaces synchronous operations by req/ack.
$\rightsquigarrow$ Translation difficult to formalise directly on the syntax.

Related work & perspectives.

Related work.

- Other standard models of game semantics [Lai05, ST17].
  $\rightsquigarrow$ Only results for may-testing.
- Models of [EHS13], fully abstract for fair-testing
  $\rightsquigarrow$ No representation of the causal behaviour.
- Abstract synchronous game semantics [Mel19].
  $\rightsquigarrow$ No interpretation of languages.

Perspectives.

- Strategies offer **causal normal forms** of processes,
  Processes offer a **syntax** to represent strategies.
- Extend the result to more expressive calculus:
  - Get rid of the linearity constraint (allow initialisers)
  - Step out of the race-free setting.

Samson Abramsky, Radha Jagadeesan, and Pasquale Malacaria.
Full abstraction for PCF.
Information and Computation, 163(2):409–470, 2000.

Clovis Eberhart, Tom Hirschowitz, and Thomas Seiller.
Fully-abstract concurrent games for pi.
CoRR, abs/1310.4306, 2013.

Dan R. Ghica and Mohamed Nabih Menaa.
Synchronous game semantics via round abstraction.
In Martin Hofmann, editor, *Foundations of Software Science and Computational Structures - 14th International Conference, FOSSACS 2011, Held as Part of the Joint European Conferences on Theory and Practice of Software, ETAPS 2011, Saarbrücken, Germany, March 26-April 3, 2011. Proceedings*, volume 6604 of *Lecture Notes in Computer Science*, pages 350–364. Springer, 2011.

Martin Hyland and Luke Ong.
On full abstraction for PCF.
Information and Computation, 163:285–408, 2000.

Kohei Honda.
Types for dyadic interaction.
In Eike Best, editor, *CONCUR '93, 4th International Conference on Concurrency Theory, Hildesheim, Germany, August 23-26, 1993, Proceedings*, volume 715 of *Lecture Notes in Computer Science*, pages 509–523. Springer, 1993.

Jim Laird.
A game semantics of the asynchronous *pi*-calculus.
In *CONCUR 2005 - Concurrency Theory, 16th International Conference, CONCUR 2005, San Francisco, CA, USA, August 23-26, 2005, Proceedings*, pages 51–65, 2005.

Paul-André Melliès.
Categorical combinatorics of scheduling and synchronization for games and strategies.
In *Accepted for publication at POPL'19*, 2019.

Paul-André Melliès and Samuel Mimram.
Asynchronous games: Innocence without alternation.
In *CONCUR 2007 - Concurrency Theory, 18th International Conference*, pages 395–411, 2007.

Silvain Rideau and Glynn Winskel.
Concurrent strategies.
In *Proceedings of the 26th Annual IEEE Symposium on Logic in Computer Science, LICS 2011, June 21-24, 2011, Toronto, Ontario, Canada*, pages 409–418, 2011.

Ken Sakayori and Takeshi Tsukada.
A truly concurrent game model of the asynchronous π-calculus.

In *Proceedings of the 20th International Conference on Foundations of Software Science and Computation Structures - Volume 10203*, pages 389–406, New York, NY, USA, 2017. Springer-Verlag New York, Inc.